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Distribution in a Neoclassical Growth Model**

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Heterogeneous Conformism and Wealth Distribution in a Neoclassical Growth Model*

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Abstract

This paper explores the role of consumption externalities in a neoclassical growth model in which households have heterogeneous preferences. We find that a higher degree of average conformism accelerates the convergence speed of the economy towards the steady state as in the case of homogeneous conformism. Furthermore, we reveal that the wealth inequality expands or shrinks in the case of heterogeneous conformism, while it does not expand but shrinks in the case of homogeneous conformism.

Keywords: consumption externalities, heterogeneous agents, wealth distribution

JEL Classification Code: D31, E13, E21, O40

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1 Introduction

It has long been recognized that social comparison is one of the central features of human behavior. In recent years, a number of experimental studies challenge to investigate whether social comparison affects individual well-being.¹ Along with the development in the neurosciences and behavioral economics, there has been a renewed interest in the role of consumption externalities in macroeconomic dynamics. The basic assumption of this literature is that consumers' felicity depends not only on their private consumption but also on the average consumption in the economy at large. The presence of such a psychological external effect may alter saving behaviors of consumers and thus dynamic property of the entire economy. Based on this idea, the foregoing studies have discussed various issues such as asset pricing (Abel 1990 and Galí 1994), income taxation (Ljungqvist and Uhlig 2000 and Fisher and Hof 2000), equilibrium efficiency (Liu and Turnovsky 2005, Nakamoto 2009 and Arrow and Dasgupta 2009), belief-driven business cycles (Alonso-Carrera, et al. 2008, Chen and Hsu 2007, Chen et al. 2013 and 2014, and Weder 2000) and long-term economic growth (Carroll et al. 1997 and 2000, and Harbaugh 1996).²

While the existing macroeconomic studies on consumption externalities discuss a variety of topics, they share a common feature: all the studies mentioned above employ representative-agent models. In the representative-agent economy, the social average consumption coincides with the level of private consumption and, hence, consumption behavior of all the agents are identical. As a result, the existing studies employing the representative-agent models fail to capture the social comparison behavior of households in a satisfactory manner. In this respect, García-Peñalosa and Turnovsky's (2008) study is a notable exception. These authors contrast a neoclassical growth model with consumption externalities in which asset holdings of households are heterogeneous so that consumption behavior of each agent is divergent from

¹For example, Fliessbach et al. (2007) examine the impact of social comparison on brain activity using functional magnetic resonance imaging (fMRI), showing that not only the absolute level of payment but also relative level of payment similarly affect brain activity. In their paper, it is considered that neurophysiological evidence supports the importance of social comparison in the human brain. Using survey-experimental methods, Alpizar et al (2005) show that, on average, both absolute and relative consumption matter for individual well-being, and conclude that most individuals are interested in others' consumption behavior.

²Other influential studies on economic analyses of consumption externalities include Carlsson et al. (2007), Clark et al. (2008), Dupor and Liu (2003), Easterlin (2001), Frank (2005) and Luttmer (2005).

each other.³ Their main concern is to explore how the presence of consumption externalities affects the pattern of wealth distribution and the transition dynamics of the aggregate economy. Although García-Peñalosa and Turnovsky (2008) make a substantial extension, they still assume that households have identical, homothetic preference. Due to this assumptions, the aggregate behavior of the economy is independent of wealth distribution.

The purpose of this paper is to extend García-Peñalosa and Turnovsky (2008) by introducing heterogeneous preferences into their setting.⁴ We are particularly concerned with the situation where households have different degree of conformism in the sense that every household likes being similar to others, but the degree of such an enthusiasm is agent specific. As shown in next section, the degree of conformism is determined by the strength of consumption external effect as well as by the intertemporal elasticity of private consumption. It is to be pointed out that a number of behavioral economics studies emphasize that behavior of social comparison is heterogeneous among consumers depending on the agents' characteristics such as income, age, race, gender, family status, education, occupation and urbanity: see, for example, Burns (2006), Hewstone et al. (2002), Maurer and Meier (2008), Mullen et al. (1992), and Rubin and Willis (2002). Our study follows such a research agenda

The key feature of our generalization is that we can distinguish the effects of average degree of consumption externalities from those of individual level of consumption conformism. We show that the average degree of households' conformism plays a relevant role in determining the behavior of the economy at large, while individual degrees of conformism yield a decisive impact on wealth distribution. More specifically, we present three findings. First, an economy with a higher degree of conformism grows faster than an economy with a lower degree of conformism. Namely, a rise in the average level of consumer conformism increases the converging speed of the aggregate economy. Second, a household with a high degree of individual conformism accumulates her wealth faster than a household whose conformism is weak. As a result, an initially poor household may catch up with an initially rich household, if the poor has a strong conformism in her consumption behavior. Third, the presence of

³It is to be noted that Koyuncu and Turnovsky (2010) examine the effects of income taxation in the model of García-Peñalosa and Turnovsky (2008).

⁴While García-Peñalosa and Turnovsky (2008) assume that labor supply is variable, we treat a model with fixed labor supply. In this point, their model is more general than our formulation.

heterogeneous conformism may enhance wealth inequality in the long-run equilibrium. This result is in contrast to the conclusion of García-Peñalosa and Turnovsky (2008) who show that consumption externalities tend to reduce the inequality of wealth distribution if households' preferences are identical and homothetic. We present the conditions under which the presence of consumption externalities enlarges wealth inequality in the steady state. We discuss our findings based on the general form of utility function as well as on an specific form of utility function that is frequently employed in the macroeconomics literature on consumption externalities. Finally, our numerical examples show that these findings can be seen under the plausible parameter set.

The reminder of this paper is organized as follows. Section 2 sets up the baseline framework. Section 3 characterizes the steady-state equilibrium and the equilibrium dynamics of the aggregate economy. Section 4 discusses the effects of consumption externalities on the dynamic behavior relative wealth and the wealth distribution in the steady-state equilibrium. Section 5 concludes.

2 Baseline Setting

2.1 Production and Consumption

We consider a simple neoclassical growth model with identical firms. The aggregate production function is assumed to satisfy constant returns to scale with respect to capital and labor, and it is expressed as

$$Y = F(\hat{K}, L) = Lf(K)$$

where Y is output, \hat{K} is capital, L is labor and $K \equiv \hat{K}/L$ denotes capital intensity. The productivity function, $f(K)$, is monotonically increasing, strictly concave in K and satisfies the Inada conditions. In competitive factor and final good markets, the real rent and real wage rate are respectively determined by

$$r = f'(K) = r(K), \quad w = f(K) - Kf'(K) = w(K). \quad (1)$$

There is a continuum of households with a unit measure. Households are assumed to be heterogeneous in the sense that each household has agent-specific preferences and different

stock of wealth. The instantaneous utility function of type i household is

$$u^i = u^i(c_i, C), \quad i \in [0, 1].$$

Here, c_i denotes private consumption of type i household and C is the average consumption in the economy at large:

$$C = \int_0^1 c_i di. \quad (2)$$

The above formulation means that an individual household's felicity is affected by the presence of consumption externalities represented by the average consumption of an entire economy.⁵ In what follows, we assume that $u^i(c_i, C)$ is monotonically increasing and strictly concave in c_i . We also assume that $u^i(c_i, C)$ is a monotonic function of C . Further restrictions on the individual utility function are discussed in Section 2.3.

The i -th agent maximizes a discounted sum of utilities

$$U^i = \int_0^\infty e^{-\rho t} u^i(c_i, C) dt$$

subject to flow budget constraint

$$\dot{a}_i = r a_i + w l_i - c_i, \quad (3)$$

where a_i and l_i respectively denote wealth holding and labor supply. The initial holding of wealth $a_i(0)$ is given and each household is subject to the non-Ponzi game condition such that

$$\lim_{t \rightarrow \infty} \exp\left(-\int_0^t r_s ds\right) a_i \geq 0. \quad (4)$$

When solving this problem, the household takes the entire sequence of the reference consumption, $\{C(t)\}_{t=0}^\infty$, as given.

Denoting the (private) utility price of capital by q_i , the optimization conditions give the following:

$$u_1^i(c_i, C) = q_i, \quad (5)$$

$$\dot{q}_i = q_i(\rho - r), \quad (6)$$

⁵In a general setting, the felicity function is given by $u_i = u^i(c_i, C_i)$. Here, C_i denotes the average consumption in a i -th group of agents, that is, $C_i = \int_{i \in N_i} c_i di$, where $N_i \subset [0, 1]$ is a subset of agents. In this paper we focus on the simplified case where external effects prevail the entire economy.

together with the transversality condition:

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_i a_i = 0. \quad (7)$$

We assume that each household supplies one unit of labor in each moment of time so that $l_i = 1$. Since the mass of households is unity, the aggregate labor is also $L = 1$. The net wealth of this economy is the aggregate capital stock, and thus the equilibrium condition of the asset market is given by

$$K = \int_0^1 a_i di.$$

Since K is only real asset, we may assume that households directly own real capital, so that we set $a_i = k_i$ in the subsequent discussion.

Finally, the equilibrium condition of the final good market is

$$Y = \dot{K} + C. \quad (8)$$

For notational simplicity, we ignore capital depreciation.

2.2 Characterizing Competitive Equilibrium

We have assumed that the households have heterogeneous preferences, so that the dynamic behaviors of the aggregate consumption, C , and capital, K , are not independent of the behaviors of individual variables, c_i and k_i . Moreover, the households constitute a continuum, meaning that we should treat a dynamic system that involves an infinite number of endogenous variables. Hence, it is not trivial to confirm whether we can obtain a well-defined, tractable dynamic system that characterizes the perfect-foresight competitive equilibrium of our economy. In this respect, it is useful to analyze a pseudo planning economy whose behavior mimics the decentralized economy. In what follows, we set up a pseudo planning problem whose solution exactly corresponds to the competitive equilibrium of our model economy. Then we show that the solution of the planning problem provides us with a well defined dynamic system.

Denoting by ω_i a weight of individual i 's utility, we assume that the planner solves the following problem:

$$\max_{\{c_i\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \left[\int_0^1 \omega_i u^i(c_i, C) di \right] dt, \quad \omega_i > 0, \quad i \in [0, 1]$$

subject to the resource constraint

$$\dot{K} = f(K) - \int_0^1 c_i di$$

and a given initial level of aggregate capital, $K(0)$. In this problem the planner maximizes a weighted sum of individual welfare. Here, the key assumption is that in solving this problem, the planner takes the external effect, i.e. the sequence of aggregate consumption, $\{C_t\}_{t=0}^\infty$, involved in the individual utility function as given.

To solve the problem, we set up the Hamiltonian function in such a way that

$$H = \int_0^1 \omega_i u^i(c_i, C) di + \lambda \left[f(K) - \int_0^1 c_i di \right],$$

where λ denotes the shadow value of aggregate capital evaluated by the social welfare. The optimization conditions include

$$\omega_i u_1^i(c_i, C) = \lambda, \quad (9)$$

$$\dot{\lambda} = \lambda(\rho - f'(K)), \quad (10)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) K(t) = 0. \quad (11)$$

Due to the assumption of strict concavity of $u^i(\cdot)$ with respect to c_i , condition (9) shows that the optimal level of consumption of agent i is uniquely written as

$$c_i = c^i\left(\frac{\lambda}{\omega_i}, C\right), \quad (12)$$

where the consumption demand function $c^i(\cdot)$ monotonically decreases with λ/ω_i . The reduced form of a complete dynamic system for this planning problem is thus given by

$$\begin{aligned} \dot{K} &= f(K) - \int_0^1 c^i\left(\frac{\lambda}{\omega_i}, C\right) di, \\ \dot{\lambda} &= \lambda(\rho - f'(K)). \end{aligned}$$

In addition, aggregation of individual consumption demand yields:

$$\int_0^1 c^i\left(\frac{\lambda}{\omega_i}, C\right) di = C. \quad (13)$$

As shown in Section 2.3, when households are conformists, their consumption demand monotonically increases with the average consumption, C . We will focus on this case and assume that

$$\int_0^1 \frac{\partial c^i(c_i, C)}{\partial C} di < 1.$$

As discussed below, in this paper we treat the case where every household is conformist: the household increases its private consumption, c_i , as the reference level of social consumption, C , rises. Thus the above restriction means that household's average conformism is not strong enough to make the aggregate consumption responded more than the rise in the reference level of consumption. Given this assumption, (13) yields a monotonic, negative relation between C and λ under a given welfare weight profile, $\{\omega_i\}_{i=0}^1$. We express such a relation as

$$C = C(\lambda), \quad C'(\lambda) < 0.$$

Consequently, the aggregate behavior of the planning economy is described by

$$\dot{\lambda} = \lambda [\rho - f'(K)], \quad (14)$$

$$\dot{K} = f(K) - C(\lambda), \quad (15)$$

together with a given $K(0)$ and the transversality condition: $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) K(t) = 0$. Since the aggregate dynamic system derived above is essentially the same as that of the standard one-sector optimal growth model, the planning problem has a unique optimal path that converges to the steady state equilibrium.

Now define

$$\frac{\lambda}{\omega_i} \equiv q_i, \quad i \in [0, 1], \quad (16)$$

which evaluates the marginal value of capital from agent i 's private perspective, that is $q_i/q_j = \omega_j/\omega_i$ so that q_i/q_j stays constant over time. Furthermore, since $\dot{q}_i/q_i = \dot{\lambda}/\lambda = \rho - f'(K)$ for all $i \in [0, 1]$, it holds that from (16) ω_i is a constant weight. As to the transversality condition, (7) and $q_i \omega_i = \lambda$ yield

$$\frac{1}{\omega_i} \lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) k_i(t) = 0.$$

Aggregating both sides of the above gives

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) K(t) = 0.$$

Therefore, the transversality condition corresponding planning problem is satisfied as well. In view of the equilibrium condition of the final good market and the determination of factor prices, we see that the aggregate behavior of the competitive economy mimics the optimal trajectory of the pseudo planning economy defined above. Therefore, the aggregate behavior

of our economy is completely characterized by a pair of differential equations of K and λ given by (14) and (15).

Finally, to complete our analysis we should determine the welfare weight, ω_i . Notice that the q_j/q_i stays constant over time for all $i, j \in [0, 1]$. As a result, at the outset of planning it holds that

$$\frac{u_1^j(c_j(0), C(0))}{u_1^i(c_i(0), C(0))} = \frac{q_j(0)}{q_i(0)} = \frac{\omega_i}{\omega_j} \quad \text{for all } i, j \in [0, 1]. \quad (17)$$

The optimal choice of the initial consumption levels, $c_i(0)$ is determined to make the optimal trajectory starting from the initial consumption, and satisfies the intertemporal budget constraint such that

$$\int_0^\infty \exp\left(-\int_0^t r(s) ds\right) c_i(t) dt = k_i(0) + \int_0^\infty \exp\left(-\int_0^t r(s) ds\right) w(t) dt, \quad i \in [0, 1],$$

where $r(t) = f'(K(t))$ and $w(t) = f(K_t) - f'(K(t))K(t)$.⁶ Given the initial holding of capital, $k_i(0)$, the level of $c_i(0)$ is uniquely determined, so that $C(0) = \int_0^1 c_i(0) di$ also takes a unique value. Hence, if ω_i ($i \in [0, 1]$) is selected to satisfy (17), then the solution of the pseudo-planning problem coincides with the competitive equilibrium.

Note that q_i is proportional to λ and from (12), c_i depends on C and q_i . In addition, the dynamic behavior of k_i depends on K and c_i . Therefore, once the optimal path of (K, C, λ) in the planning problem are established, behaviors of q_i , c_i and k_i are determined as well.

2.3 Conformism and Consumption Behavior

The conditions (5) and (6) yield

$$\dot{c}_i = -\frac{u_1^i(c_i, C)}{u_{11}^i(c_i, C)}(r - \rho) - \frac{u_{12}^i(c_i, C)}{u_{11}^i(c_i, C)}\dot{C}.$$

We express this equation as

$$\dot{c}_i = \phi_i(c_i, C)(r - \rho) + \Lambda_i(c_i, C)\dot{C}, \quad (18)$$

where

$$\phi_i(c_i, C) = -\frac{u_1^i(c_i, C)}{u_{11}^i(c_i, C)} > 0, \quad (19a)$$

⁶The intertemporal budget constraint holds as an equality due to the non-Ponzi-game constraint and the transversality condition.

$$\Lambda_i(c_i, C) = -\frac{u_{12}^i(c_i, C)}{u_{11}^i(c_i, C)}. \quad (19b)$$

In the above, $1/\phi_i(c_i, C)$ represents the degree of absolute risk aversion of type i household. Following Gollier (2004), we call $\Lambda_i(c_i, C)$ the *degree of conformism* of type i household. This function shows how the private consumption responds to a change in the average consumption to keep the marginal utility of private consumption constant. If $\Lambda_i > 0$, the household i is a *conformist* in the sense that she changes her own consumption in the same direction of the change in the average consumption. In contrast, if $\Lambda_i(c_i, C) < 0$, the household changes her consumption in the opposite direction: the household is an anti-conformist. Moreover, when $\Lambda_i(c_i, C) > 1$, the household is an over-conformist, because the household changes her consumption more than a change in the average consumption to keep her marginal utility of private consumption constant. In this paper we focus on the case each household is a conformist but is not over-conformist, so that we assume $0 < \Lambda_i(c_i, C) < 1$ for all $i \in [0, 1]$.

The specification of the utility function that has been frequently used in the literature is the following multiplicative form of externalities:⁷

$$u^i(c_i, C) = v_i(c_i) \eta_i(C), \quad i \in [0, 1],$$

where $v_i'(c_i) > 0$, $v_i''(c_i) < 0$ and $\eta_i'(C) > 0$. Given this functional form, we obtain

$$\phi_i(c_i) = -\frac{v_i'(c_i)}{v_i''(c_i)} > 0, \quad \Lambda_i(c_i, C) = \phi_i(c_i) \frac{\eta_i'(C)}{\eta_i(C)} > 0.$$

Note that if the external effects are introduced in the multiplicative form, the absolute risk aversion depends only on the private consumption, while the degree of conformism is affected by private as well as social level of consumption (the case of *non-separable conformism*).⁸

A simple example is to set $v_i(c_i) = c_i^{1-\gamma}/(1-\gamma)$ and $\eta_i(C) = C^{-\theta_i(1-\gamma)}$, so that the instantaneous utility function is:

$$u^i(c_i, C) = \frac{(c_i C^{-\theta_i})^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad \gamma \neq 1, \quad 0 < \theta_i < 1. \quad (20)$$

⁷For instance, this type of utility function is given in Gali (1994) and Carroll et al (1997).

⁸In contrast to the non-separable conformism, we can see the separable conformism in the sense that the degree of conformism depends on the average consumption alone by using the subtractive form of consumption externalities as follows:

$$u^i(c_i, C) = \frac{(c_i - \eta_i(C))^{1-\beta}}{1-\beta}, \quad \beta \neq 1, \quad \beta > 0.$$

where β is the degree of risk aversion.

where γ shows the common degree of absolute risk aversion among households and θ_i is the degree of external effect of type i household.⁹ In this specification, the heterogeneity of preferences only stems from the difference in θ_i . Here, we obtain

$$\phi_i(c_i, C) = \frac{c_i}{\gamma}, \quad \Lambda_i(c_i, C) = \left(1 - \frac{1}{\gamma}\right) \theta_i \frac{c_i}{C},$$

implying that if the household is a conformist, we should assume $\gamma > 1$ because $0 < \theta_i < 1$. Hence, in this well employed functional form, the degree of individual conformism depends on the intertemporal elasticity of private consumption, $1/\gamma$, the individual degree of external effect, θ_i , as well as on the private consumption relative to the social average, c_i/C . Notice that even if the preference parameters are identical ($\theta_i = \theta$ for all i), the degree of individual conformism may differ each other unless $c_i = C$ for all i . It is also to be noted that in this example the Euler equation (18) is rewritten as¹⁰

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\gamma} (r - \rho) + \left(1 - \frac{1}{\gamma}\right) \theta_i \frac{\dot{C}}{C},$$

which leads to

$$\frac{\dot{c}_i}{c_i} - \frac{\dot{c}_j}{c_j} = \left(1 - \frac{1}{\gamma}\right) (\theta_i - \theta_j) \frac{\dot{C}}{C}.$$

As a result, under the conditions of $\gamma > 1$ and $\theta_i > 0$, while each household changes her consumption in the same direction of the change in the average consumption, the relative speed of consumption changes between two individuals depends on the ranking of the degree of external effects, that is, the sign of $\theta_i - \theta_j$. Since the speed of consumption adjustment affects the speed of wealth accumulation, the example clearly demonstrates that the heterogeneity of consumption conformism may yield a decisive effect on the long-run wealth distribution among households.

⁹The introduction of heterogeneous risk aversion means that the economy has two types of conformism: $\theta_i > 0$ and $(1 - 1/\gamma_i) > 0$ so that $(1 - 1/\gamma_i)\theta_i > 0$; $\theta_i < 0$ and $(1 - 1/\gamma_i) < 0$ so that $(1 - 1/\gamma_i)\theta_i > 0$. As shown later, since the key element of seeing our findings is the degree of conformism, the introduction of heterogeneous risk aversion may make our paper verbose in the sense that the above two types of conformism lead to the same findings. Therefore, we omit the heterogeneity of risk aversion.

¹⁰If the heterogeneity of risk aversion is also present, then the consumption growth path is more complicated in the sense that the growth rate of private consumption without the external effect is different among households.

3 The Aggregate Economy

3.1 Aggregate Dynamics and the Steady-State Equilibrium

As shown in the previous section, the aggregate dynamics of our economy are described by the total capital, K , and its utility price, λ . Since the aggregate consumption, C , is a function of λ , the aggregate dynamics can be considered in terms of K and C as well. To see this, it is to be noted that from (18), the average (aggregate) consumption follows:

$$\dot{C} = (r - \rho) \int_0^1 \phi_i(c_i, C) di + \left(\int_0^1 \Lambda_i(c_i, C) di \right) \dot{C},$$

which leads to

$$\dot{C} = \Delta (r - \rho), \quad \Delta \equiv \frac{\int_0^1 \phi_i(c_i, C) di}{1 - \int_0^1 \Lambda_i(c_i, C) di}, \quad (21)$$

where Δ/C represents the elasticity of intertemporal substitution in social consumption. In the following, we assume that the average degree of conformism of the society does not exhibit over-conformism so that¹¹

$$\int_0^1 \Lambda_i(c_i, C) di < 1, \quad (22)$$

implying that Δ has a positive value. Equation (21) shows that, other things being equal, a higher degree of average level of conformism makes the average consumption more sensitive to a change in the real interest rate, r .

Substituting (21) into (18), we find that the consumption of individual household follows

$$\dot{c}_i = \Phi_i(r - \rho), \quad i \in [0, 1], \quad (23)$$

where $\Phi_i = \phi_i(c_i, C) + \Lambda_i(c_i, C) \Delta$. This expression means that when the household of type i has a higher degree of conformism, $\Lambda^i(c_i, C)$, her private consumption is more sensitive to a change in the real interest rate.

The differences of Φ_i among households are derived by the heterogeneous conformism and

¹¹In the case of (20), (22) is given by $\int_0^1 \theta_i c_i di / C < \gamma / (\gamma - 1)$.

the differences of initial levels of capital holdings. Using (20), the Φ_i can be given by:¹²

$$\Phi_i = \frac{c_i}{\gamma \left(1 - \left(1 - \frac{1}{\gamma}\right) \Theta\right)} \left\{ 1 + \left(1 - \frac{1}{\gamma}\right) \underbrace{(\theta_i - \Theta)}_{(\#1)} \right\}, \quad (24)$$

where $\Theta = \frac{\int_0^1 \theta_i c_i di}{C}$ represents the average degree of external effect, while $\left(1 - \frac{1}{\gamma}\right) \Theta$ represents the average degree of conformism. Even if the preferences are homogeneous, (24) can be reduced to the elasticity of intertemporal substitution in the representative-agent model; alternatively, the heterogeneity of consumption externalities yields the difference of $(\theta_i - \Theta)$ in the term $(\#1)$. Noting that $\gamma > 1$, others things being equal, an increase in the degree of conformism of type i ' household $(1 - 1/\gamma)\theta_i$ and the average degree of conformism $(1 - 1/\gamma)\Theta$ leads to the increase in the value of Φ_i .

From (8) the dynamic behavior of the aggregate (average) capital follows

$$\dot{K} = f(K) - C. \quad (25a)$$

As a consequence, a complete dynamic system of the entire economy consists of (23), (25a),

$$\dot{k}_i = rk_i + w - c_i, \quad i \in [0, 1], \quad (25b)$$

together with (1), (2) and a given initial level of capital distribution among the households.

The steady-state levels of average consumption and capital stock, C^* and K^* , are uniquely determined by

$$f(K^*) = C^*, \quad (26a)$$

$$f'(K^*) = \rho. \quad (26b)$$

The above steady-state conditions demonstrate that distribution of wealth and the presence of consumption externalities fail to affect the steady-state levels of average variables as in the representative-agent model with the fixed labor supply. In addition, when all the households are conformists so that $\Lambda_i(c_i, C) > 0$ for all i , and if the the average degree of conformism

¹²The long-run level of private consumption is given by:

$$c_i(t) = c_i(0) \exp \left(\int_0^t \Phi_i(s)(r(s) - \rho) ds \right),$$

and furthermore, the initial jump of private consumption is characterized by the degree of conformism and the initial holding of capital of type i 's household.

satisfies (22), then the average Euler equation exhibits the familiar pattern of dynamics: the average consumption increases (decreases) when the average capital is lower (higher) than its steady state level, K^* . Therefore, K and C follow a stable saddle path converging to the steady state given above.

3.2 Convergence Speed

We first examine local dynamics of the aggregate system around the steady state equilibrium. The linearly approximated system of (21) and (25a) at the steady state consists of the following dynamic equations

$$\dot{K} = \rho(K - K^*) - (C - C^*),$$

$$\dot{C} = \Delta^* f''(K^*)(K - K^*),$$

where Δ^* denotes the steady state level of Δ given by the following:

$$\Delta^* = \frac{\int_0^1 \phi_i(c_i^*, C^*) di}{1 - \int_0^1 \Lambda_i(c_i^*, C^*) di} \quad (27)$$

and c_i^* denotes the steady-state level of the individual consumption.

Noting that $K^* = f'^{-1}(\rho)$ and $C^* = f(f'^{-1}(\rho))$, we can rewrite $K^* = K^*(\rho)$, $C^* = C^*(\rho)$ and $w^* = w^*(\rho)$. Therefore, the steady-state value of c_i satisfies

$$c_i^* = \rho k_i^* + w^*(\rho). \quad (28)$$

Furthermore, we find that the stable root of the above system is

$$\mu_s = \frac{1}{2} \left[\rho - (\rho^2 - 4\Delta^* f''(K^*(\rho)))^{1/2} \right] < 0, \quad (29)$$

where Δ^* is given by (27).

Since the absolute value of the stable root represents the speed of convergence on the aggregate economy on the stable saddle path, we immediately see that the economy with a higher degree of average conformism, and hence a higher value of Δ^* exhibits a higher speed of convergence towards the steady state. More specifically, the speed of convergence is faster as the positive value of $\int_0^1 \Lambda_i^* di$ approaches the unity and the positive value of $\int_0^1 \phi_i^* di$ is larger. Furthermore, and more importantly, because of the heterogeneity of conformism, the

steady-state distribution of capital itself affects the value of Δ^* . Using (20), the following holds.¹³

$$\int_0^1 \phi_i(c_i^*, C^*) di = \frac{C^*}{\gamma}, \quad \int_0^1 \Lambda_i(c_i^*, C^*) di = \left(1 - \frac{1}{\gamma}\right) \frac{\int_0^1 \theta_i c_i^* di}{C^*}. \quad (30)$$

As seen in (30), the value of $\int_0^1 \phi_i(c_i^*, C^*) di$ is uniquely given, but the value of $\int_0^1 \Lambda_i(c_i^*, C^*) di$ is affected by the wealth distribution. In detail, looking at $\int_0^1 \theta_i c_i^* di$, we can argue that when the relatively wealthier households hold stronger degrees of conformism, the value of $\int_0^1 \theta_i c_i^* di$ and hence $\int_0^1 \Lambda_i(c_i^*, C^*) di$ are greater so that the speed of convergence in the unequal economy becomes faster.

Dynamic behaviors of individual consumption and wealth are described by

$$\begin{aligned} \dot{k}_i &= rk_i + w - c_i = f'(K)(k_i - K) + f(K) - c_i, \\ \dot{c}_i &= [\phi_i(c_i, C) + \Lambda_i(c_i, C)\Delta](f'(K) - \rho). \end{aligned}$$

On the stable saddle path of the aggregate system, it holds that $C - C^* = (\rho - \mu_s)(K - K^*)$. Hence, the approximated behavior of individual capital, individual consumption and the aggregate capital respectively follow

$$\begin{aligned} \dot{k}_i &= \rho(k_i - k_i^*) - (c_i - c_i^*) + f''(K^*)(k_i^* - K^*)(K - K^*), \\ \dot{c}_i &= \Phi_i^* f''(K^*)(K - K^*), \\ \dot{K} &= \mu_s(K - K^*), \end{aligned}$$

where

$$\Phi_i^* = \phi_i^* + \Lambda_i^* \Delta^* > 0. \quad (31)$$

Note that the stable root of this system is still μ_s , which means that on the stable saddle path of the entire economy each relation between individual capital (or individual consumption) and the aggregate capital satisfies

$$k_i - k_i^* = \frac{\frac{\Phi_i^* f''(K^*)}{\mu_s} - f''(K^*)(k_i^* - K^*)}{\rho - \mu_s} (K - K^*), \quad (32)$$

$$c_i - c_i^* = \frac{\Phi_i^* f''(K^*)}{\mu_s} (K - K^*). \quad (33)$$

¹³ Alternatively, using the subtract form of conformism, since the conformism is separable from the individual capital, the wealth distribution does not have any impacts on the speeds of convergence if the degrees of conformism are heterogeneous alone.

Therefore, on the approximated saddle path both individual consumption and capital move into the same direction as the aggregate capital changes. In addition, it is seen that, other things being equal, a higher level of individual conformism (a higher value of $\Lambda_i(c_i, C)$) raises the responses of c_i and k_i to a change in the aggregate capital.

To sum up, we have seen the following result as to the local dynamics of the economy:

Proposition 1 *(i) The speed of convergence of the aggregate economy increases with the degree of average conformism in the economy at large; (ii) the speed of convergence of capital and consumption of each consumer increases with her own degree of conformism; and (iii) using (20) with the heterogeneous conformism, the wealth distribution affects the speeds of convergence as seen in (30).*

Result (i) in the above proposition means that even though the degrees of conformism differ each other, we still have the same outcome established in the model with homogeneous preference: a higher degree of consumption conformism raises the convergence speed of an entire economy. Result (ii) states that the degree of conformism of each household is one of the relevant determinants of long-run wealth distribution among households. Result (iii) is a natural consequence of our setting in which the behavior of aggregate variables are not independent of wealth distribution. In sum, we have confirmed that the individual degree of conformism affects wealth distribution, which in turns yields impacts on the aggregate behavior of the aggregate economy. In the next section, we examine the relation between individual conformism and long-run wealth distribution in detail.

4 Wealth Distribution

4.1 Behaviors of Relative wealth

We now examine the role of heterogeneous conformism in determining wealth distribution at the steady state. Let us denote the relative capital holding of agent i by $\tilde{k}_i = k_i/K$. Using the capital accumulation equations (25a) and (25b), we derive the dynamics of relative wealth as follows:

$$\dot{\tilde{k}}_t = \frac{1}{K} (f'(K)K - f(K)) (\tilde{k}_i - 1) + \frac{C}{K} \left(\tilde{k}_i - \frac{c_i}{C} \right). \quad (34)$$

García-Peñelosa and Turnovsky (2008) assume that the utility function of each agent is not only identical but also homothetic both from private and social perspectives. Given those assumptions, consumption of each household changes at the same rate so that the relative consumption of each agent, c_i/C , stays constant over time and the level of c_i/C is determined by the initial distribution of wealth among the households. In contrast, the relative consumption in our model changes during the transition process, which may yield substantial effects on wealth distribution.

For the purpose of comparison, let us first consider the case of identical and homothetic preferences where c_i/C does not change over time. Observe that the relative wealth along the stable saddle path satisfies the following:¹⁴

$$\tilde{k}_i(t) = \tilde{k}_i^* + (\tilde{k}_i^* - 1)Z^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}, \quad (35)$$

where

$$Z^* = \frac{(K^* - K(0))\rho A^*}{(\rho - \mu_s)K^*}, \quad (36)$$

$$A^* = 1 + \frac{f''(K^*)K^*}{f'(K^*)} - \frac{K^* f'(K^*)}{f(K^*)} - \frac{\mu_s}{\rho} \left(1 - \frac{K^* f'(K^*)}{f(K^*)} \right).$$

Notice that Z^* and A^* depend only on the steady-state levels of aggregate variables except for the stable root μ_s . In this setting, García-Peñelosa and Turnovsky (2008) conclude that the elasticity of substitution between labor and capital in the production function, which affects the sign of A^* , is a key element when determining the wealth distribution in the steady state. To simplify our discussion, in what follows we assume that

$$1 \geq \frac{f'(K^*)K^*}{f(K^*)} - \frac{f''(K^*)K^*}{f'(K^*)}, \quad (37)$$

and, hence, A^* in (36) has a positive value. For example, if $f(K)$ is a Cobb-Douglas production function $f(K) = K^\alpha$ ($\alpha < 1$), then condition (37) is satisfied.

Using (35), we see that the difference of capital stock between the households i and j is:

$$\tilde{k}_i(t) - \tilde{k}_j(t) = (\tilde{k}_i^* - \tilde{k}_j^*) \left(1 + Z^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t} \right). \quad (38)$$

The above expression demonstrates that as long as $K(0) < K^*$, if $\tilde{k}_i^* > (<) \tilde{k}_j^*$, then $\tilde{k}_i(t) > (<) \tilde{k}_j(t)$ for all $t \geq 0$. That is, the catching-up does not arise. The intuitive explanation

¹⁴See Appendix A with respect to the derivation.

is as follows. From (5) and (6) $u_1(c_i, C)/u_1(c_j, C)$ is constant over time. Since $u_1(c_i, C)$ is monotonically decreasing in c_i for all $C (> 0)$, if $c_i(0) > c_j(0)$, then $c_i(t) > c_j(t)$ for all $t > 0$. In view of the intertemporal budget constraint for individual household, the identical preference mean that if $\tilde{k}_i(0) > \tilde{k}_j(0)$, then $c_i(0) > c_j(0)$. As a consequence, if the initial capital distribution satisfies that $\tilde{k}_i(0) > \tilde{k}_j(0)$, then it holds that $\tilde{k}_i^* > \tilde{k}_j^*$ and $c_i^* > c_j^*$. Namely, regardless of the presence of consumption externalities, the initial pattern of wealth distribution is kept in the long run equilibrium.

Now consider the case of heterogeneous preferences. In our general setting, while the relative marginal utility of private consumption, $u_1^i(c_i, C)/u_1^j(c_j, C)$, stays constant over time, c_i/c_j generally changes during the transition. The relative wealth in our setting is given by

$$\tilde{k}_i(t) = \tilde{k}_i^* + Z_i^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}, \quad (39)$$

where

$$Z_i^* = \frac{B^*(\tilde{k}_i^* - 1)}{K^*} + \frac{\rho - \mu_s}{K^*} \left(1 - \frac{\Phi_i^*}{\Delta^*} \right), \quad (40)$$

$$B^* = f''(K^*)K^* + \rho - \mu_s (> 0).$$

In the above, Φ_i^* is given by (31). It is to be noted that the sign of B^* is positive under (37). In view of (39), we find that the difference in capital stock between the households i and j under the heterogeneous preferences is:

$$\tilde{k}_i(t) - \tilde{k}_j(t) = \tilde{k}_i(0) - \tilde{k}_j(0) + \frac{(Z_i^* - Z_j^*)(e^{\mu_s t} - 1)(K^* - K(0))}{\rho - \mu_s}. \quad (41)$$

Here, the term $(Z_i^* - Z_j^*)$ stems from the presence of heterogeneous conformism. If this expression shows that $Z_i^* < Z_j^*$, then $\tilde{k}_i(0) > \tilde{k}_j(0)$ does not necessarily establish $\tilde{k}_i^* > \tilde{k}_j^*$. If the initially less wealthy household j catches up with the initially richer household i at time \hat{t} , then we can show that

$$\hat{t} = \frac{1}{\mu_s} \log \left(\frac{\tilde{k}_j(0) - \tilde{k}_i(0) + \frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}}{\frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}} \right). \quad (42)$$

The catching-up arises if and only if \hat{t} has a positive value. More specifically, we conclude:

Proposition 2 *Suppose that $K^* > K(0)$ and $k_i(0) > k_j(0)$. Then, (i) the initially poorer household j will never catch up with the other under the identical and homothetic preferences;*

and (ii) the initially poorer household j will (will not) catch up in wealth if the following inequality is satisfied:

$$\frac{\Phi_j^* - \Phi_i^*}{\Delta^*} > (<) \frac{k_i(0) - k_j(0)}{1 - K(0)/K^*} + \frac{A^*(\tilde{k}_j^* - \tilde{k}_i^*)}{(\rho - \mu_s)}. \quad (43)$$

Proof. Since the catching-up arises if and only if $\hat{t} > 0$, from (42) we can derive the following:

$$0 < \frac{\tilde{k}_j(0) - \tilde{k}_i(0)}{\frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}} + 1 < 1,$$

which leads to the condition (43) with respect to the catching-up. ■

The condition (43) shows that the catching-up would occur if and only if an initially poorer agent j has a sufficiently large value of Φ_j^* . This is plausible because the greater elasticity of intertemporal substitution means that the household plans to increase own savings, which leads to a higher level of wealth in the future.

To combine this finding with the heterogeneous conformism, let us consider (20). For expositional simplicity, we assume that the initial holdings of capital satisfy $k_i(0) > k_j(0)$ and it holds that $k_i^* = k_j^*$ (so that $c_i^* = c_j^*$). Given these conditions, we find:

$$\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_j^*, C^*)}{\Delta^*} = \frac{\left(1 - \frac{1}{\gamma}\right)}{C^*} (\theta_j - \theta_i),$$

Thus if households i and j have the same magnitude of conformism such that $\theta_i = \theta_j$, it holds that $\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_j^*, C^*)}{\Delta^*} = 0$, thereby being unable to see the catching-up (i.e., Proposition 2(i)). Next, if the degrees of conformism between households i and j differ each other, then $\Phi_j^* - \Phi_i^*$ has a positive value if and only if $\theta_j > \theta_i$, which means that the catching-up may arise. The above condition means that if household j has a stronger conformism towards the social average consumption than household i , the initial discrepancy of consumption and wealth may be eliminated in the long run. In words, these results suggest that if an initially poor household who has smaller wealth than the social average level has a strong aspiration of catching up the social average, she may overtake the wealth held by an initially rich household with a weak level of consumption conformism.

4.2 Patterns of Wealth Distribution

In this subsection we consider the dynamics of wealth distribution. Defining the difference between the aggregate and the individual capital stocks as $\sigma_i(t) \equiv \tilde{k}_i - 1$, we rewrite (39) as

$$\sigma_i(t) = \sigma_i^* + Z_i^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}. \quad (44a)$$

Again, we assume that the initial level of aggregate capital satisfies $K(0) < K^*$. We first examine the case of identical and homothetic preferences. In this case, from (35), equation (44a) is rewritten as

$$\sigma_i(t) = \sigma_i^* \left(1 + \frac{Z^* (K^* - K(0))}{\rho - \mu_s} e^{\mu_s t} \right). \quad (44b)$$

Equation (44b) shows the characteristics of dynamics of relative wealth under the identical wealth. Differentiating (44b) with respect to time yields:

$$\dot{\sigma}_i(t) = \frac{\mu_s \sigma_i^* Z^* (K^* - K(0))}{\rho - \mu_s} e^{\mu_s t}.$$

Then, we can see that $\dot{\sigma}_i(t) > (<) 0$ if $\sigma_i^* < (>) 0$ for all households, so that the dispersion in wealth holdings shrinks over time under our assumption (37). For example, there is the relative-wealth rich such as $\sigma_i^* > 0$ in the long run. Since $\dot{\sigma}_i(t) < 0$, the divergence between the level of individual wealth and the average wealth decreases over time, and $\sigma_i(t)$ converges $\sigma_i^* (> 0)$, implying that $\sigma_i(0) > \sigma_i^* > 0$. Similarly, if $\sigma_i^* < 0$ in the long run, the reverse can be applied so that the relative wealth becomes small along time, $0 > \sigma_i^* > \sigma_i(0)$. These results mean that if we define the index of wealth inequality in time t by

$$S = \int_0^1 \sigma_i^2 di,$$

then in the case of identical and homothetic preferences, the steady state level of $S^* = \int_0^1 (\sigma_i^*)^2 di$ is less than its initial level, $S(0) = \int_0^1 \sigma_i(0)^2 di$ as long as (37) holds.¹⁵

As shown in Proposition 1, the presence of consumption conformism raises the speed of convergence of the aggregate economy. Hence, when the aggregate capital increases towards

¹⁵As can be easily predicted, it holds that $S^* < S(0)$ in the identical preferences:

$$S^* - S(0) = S(0) \left(\frac{-A^*(K^* - K(0)) (2(\rho - \mu_s) + A^*(K^* - K(0)))}{(\rho - \mu_s + A^*(K^* - K(0)))^2} \right) (< 0),$$

where we raise both sides of (44b) to the double power and take account of $t = 0$.

its steady state level, the rate of return to capital declines faster in the economy with consumption externalities than in the economy without them. Note that in our model with fixed labor supply, the income differences among households only come from their capital income, $r(K)k_i$. This means that the negative impact generated by the decrease in the rate of return to capital is higher for the households whose capital holdings are larger. As a consequence, in the presence of consumption conformism, the discrepancy in capital holdings shrinks faster, so that the steady-state distribution of capital among households is more equal than in the economy without conformism. This result is a key finding of García-Peñelosa and Turnovsky (2008).

In the case of heterogeneous conformism, $\sigma_i(t)$ changes according to

$$\dot{\sigma}_i(t) = \frac{\mu_s(K^* - K(0))Z_i^*}{\rho - \mu_s} e^{\mu_s t}. \quad (45)$$

In this case, the sign of σ_i^* fails to specify the sign of $\sigma_i(t)$ during the transition. Equation (45) states that if we focus on the case $K(0) < K^*$, then

$$\text{sign } \dot{\sigma}_i(t) = -\text{sign } Z_i^*.$$

Turning back to the definition of Z_i^* given in (40), we can see that

$$Z_i^* = \underbrace{\frac{B^* \sigma_i^*}{K^*}}_{(\#2)} + \frac{\rho - \mu_s}{K^*} \left(1 - \frac{\Phi_i^*}{\Delta^*} \right). \quad (46)$$

Then, irrespective of the long-run position of individual capital, the effect in $(\#2)$ is negative (positive) if $\sigma_i^* < (>)0$, which implies that this effect makes the dispersion of relative wealth decreased. Based on the sign of $(\#2)$ in (46), the following results immediately follow:

Proposition 3 *Under the assumptions of (37) and $K(0) < K^*$, it holds that (i) if $\sigma_i^* < 0$ and $\Phi_i^*/\Delta^* > 1$, then $\dot{\sigma}_i(t) > 0$ for all $t \geq 0$; and (ii) if $\sigma_i^* > 0$ and $\Phi_i^*/\Delta^* < 1$, then $\dot{\sigma}_i(t) < 0$ for all $t \geq 0$.*

Proof. From (46) if $\sigma_i^* < 0$ and $\Phi_i^*/\Delta^* > 1$, it holds that $Z_i^* > 0$, implying that $\dot{\sigma}_i(t) < 0$ in (39). If $\sigma_i^* > 0$ and $\Phi_i^*/\Delta^* < 1$, the opposite outcome holds. ■

This proposition presents a set of conditions under which the divergence between individual capital holding and the average stock of capital monotonically decreases during the

transition as in the case of homogeneous conformism. The key element is the sign of $1 - \Phi_i^*/\Delta^*$. In particular, we notice that the degree of Φ_i^* is larger as the degree of own conformism θ_i increases. Furthermore, noting that $\int_0^1 \Phi_i^* di = \Delta^*$, condition $\Phi_i^*/\Delta^* > (<)1$ means that the elasticity of intertemporal substitution in consumption from the private perspective is higher (lower) than that from social perspective. Assuming that a household holds relatively less wealth in the long run, result (i) shows that the household whose private intertemporal elasticity of substitution is relatively high accumulates her capital faster than the social average during the transition towards the steady state. As a result, the difference between her capital and the average one shrinks during the transition. Turning to result (ii), suppose that a household holds relatively more wealth in the long run, and that the household whose private intertemporal elasticity of substitution is relatively low accumulate her capital slower than the average. Then, we are able to see that the relative wealth shrinks. Therefore, a rough implication of this proposition is that if the long-run rich households have relatively low degree of private elasticity of intertemporal substitution and if the long-run poor households have relatively high value of intertemporal substitution, then the wealth distribution in the steady state is more equal than the initial distribution.

To obtain a more precise implication, it is useful to examine the definition of Φ_i^*/Δ^* such that

$$\frac{\Phi_i^*}{\Delta^*} = \left(1 - \int_0^1 \Lambda_i^* di\right) \left(\frac{\phi_i^*}{\int_0^1 \phi_i^* di} + \frac{\Lambda_i^*}{1 - \int_0^1 \Lambda_i^* di}\right).$$

This expression reveals that the relative elasticity of intertemporal substitution in consumption between the private and the social perspectives is high, if at least one of the following conditions holds: (i) the degrees of average absolute risk aversion $\int_0^1 \phi_i^* di$ and average conformism $\int_0^1 \Lambda_i^* di$ are small; and (ii) the degrees of private absolute risk aversion ϕ_i^* and conformism Λ_i^* , are large. When Φ_i^*/Δ^* takes a relatively high value, the household i attains faster accumulation of her capital than the social average.

Notice that the conditions given in Proposition 3 are not necessary but sufficient. Therefore, we may consider a more complex situation. In particular, considering that the effect (#2) is negative for expanding the wealth inequality, if the effect of key element $1 - \Phi_i^*/\Delta^*$ on the wealth distribution is the opposite with the effect (#2), and furthermore, this effect dominates the effect (#2), then the dispersion of relative wealth increases during the tran-

sition. For example, suppose that when a household is relative-wealth rich in the long run (i.e., $\sigma_i^* > 0$) where we assume $\sigma_i(0) > 0$. If $1 < \Phi_i^*/\Delta^*$ so that $Z_i^* < 0$, then we can see the enlargement of relative wealth $\dot{\sigma}_i(t) > 0$ and $0 < \sigma_i(0) < \sigma_i^*$. Importantly, such an expansion of relative wealth cannot be seen in the case of homogeneous conformism. Considering those alternative possibilities, we may present a general condition as to the wealth distribution in the steady state:

Proposition 4 *The long-run wealth inequality is larger (lower) than the initial level of inequality if the following condition is satisfied:*

$$\left(1 - \frac{K(0)}{K^*}\right)^2 \underbrace{\left(\frac{\int_0^1 (\Phi_i^*)^2 di}{\left(\int_0^1 \Phi_i^* di\right)^2} - 1\right)}_{(\#3)} + 2 \left(1 - \frac{K(0)}{K^*}\right) \underbrace{\frac{\int_0^1 \sigma_i(0) \Phi_i^* di}{\int_0^1 \Phi_i^* di}}_{(\#4)} > (<) M^* (2 + M^*) S(0). \quad (47)$$

where

$$M^* = \left(1 - \frac{K(0)}{K^*}\right) \frac{B^*}{\rho - \mu_s}. \quad (48)$$

Proof. Following Appendix B, we derive the following:

$$S^* - S(0) = \frac{-M(2 + M)S(0) + X^*}{(1 + M)^2}, \quad (49)$$

where

$$X^* = \left(\frac{\int_0^1 (\Phi_i^*)^2 di}{\left(\int_0^1 \Phi_i^* di\right)^2} - 1\right) \left(1 - \frac{K(0)}{K^*}\right)^2 + 2 \left(1 - \frac{K(0)}{K^*}\right) \frac{\int_0^1 \sigma_i(0) \Phi_i^* di}{\int_0^1 \Phi_i^* di}. \quad (50)$$

If $X^* > (<) M(2 + M)S(0)$, which corresponds to (47), then it holds that $S^* > (<) S(0)$.

■

The conditions in Proposition 4 are rather complex, but we may obtain an intuitive implication. First, (#3) shows the dispersion of degrees of conformism in the entire economy relative to the average degree of conformism. We must notice that (#3) always has a non-negative sign by the Cauchy-Schwarz inequality, which means that the heterogeneity of conformism directly expands the wealth inequality regardless of any spatial arrangement of dispersed heterogeneous conformism. Moreover, when the degrees of conformism are largely dispersed, which corresponds to a larger value of $\int_0^1 (\Phi_i^*)^2 di$, the wealth inequality further expands. Alternatively, taking account of the homogeneity of conformism as an extreme case, we can easily see that $\int_0^1 (\Phi_i^*)^2 di = \left(\int_0^1 \Phi_i^* di\right)^2$ so that $(\#3) = 0$.

Next, consider the term (#4), which indicates to a correlation between the initial holdings of capital stocks and the heterogeneity of conformism. Assume a growing economy in the sense that $K^* > K(0)$. The effect (#4) shows that when the initial riches have the greater degrees of conformism, it holds that $\int_0^1 \sigma_i(0) \Phi_i^* di$ has a positive sign, which means that the wealth inequality tends to expand. Intuitively, when the initial riches have the greater degrees of conformism, they like to save but the initial wealth-poor people dislike the saving, which implies that the initial riches hold more wealth over time; and hence, the wealth inequality expands. On the other side, when the initially-wealth poor people have the greater degrees of conformism, they want to save over time, thereby seeing that the wealth inequality does not expand but shrink.

To examine $\int_0^1 \sigma_i(0) \Phi_i^* di$ further, we make use of the specified utility function (20):¹⁶

$$\int_0^1 \Phi_i^* \sigma_i(0) di = \frac{\rho}{\gamma} \left(\int_0^1 \sigma_i(0) \sigma_i^* di + \frac{(\gamma-1)/\gamma}{1 - \frac{\gamma-1}{\gamma C} \int_0^1 \theta_i c_i^* di} \left(\int_0^1 \sigma_i(0) \sigma_i^* \theta_i di + \int_0^1 \sigma_i(0) \theta_i di \right) \right). \quad (51)$$

In that case, it generally holds that $\int_0^1 \sigma_i(0) \sigma_i^* di > 0$, because the initially relative-wealth riches seem to keep the relative-wealth ones in the long run. Furthermore, if the rich people at the initial period have the greater degrees of conformism, we can confirm that $(\gamma-1) \left(\int_0^1 \sigma_i(0) \sigma_i^* \theta_i di + \int_0^1 \sigma_i(0) \theta_i di \right) > 0$. Hence, since $\int_0^1 \Phi_i^* \sigma_i(0) di > 0$, then wealth distribution becomes more unequal in the long run.

Although our main proposition says the possibility of increasing the wealth inequality due to the heterogeneities of conformism in an analytical way, it would be difficult to see if the wealth inequality actually increases because our model has two heterogeneities: the initial holding of capital stock and the heterogeneous conformism. Therefore, supposing that the differences of initial holdings of capital stocks among agents do not exist so that $S(0) = 0$, we pay attention to the role of heterogeneous conformism for the wealth inequality. If the utility function is identical and homothetic, the identical levels of initial capital stocks among the agents yield the identical jump of private consumption, and furthermore the relative consumption between agents is constant, thereby concluding that $S^* = S(0) = 0$.¹⁷

¹⁶To derive (51), we make use of $\int_0^1 \sigma_i(0) c_i^* di = \rho \int_0^1 \sigma_i(0) \sigma_i^* di$ and $\int_0^1 \sigma_i(0) c_i^* \theta_i di = \rho \left(\int_0^1 \sigma_i(0) \sigma_i^* \theta_i di + \int_0^1 \sigma_i(0) \theta_i di \right)$.

¹⁷When the utility function is identical and homothetic, the value of X^* is zero in (49), which can be seen that $S^* = 0$ if $S(0) = 0$.

Alternatively, when the degrees of conformism are not identical among agents, from (49), we can derive

$$S^* = \frac{\left(\frac{\int_0^1 (\Phi_i^*)^2 di}{(\int_0^1 \Phi_i^* di)^2} - 1 \right) \left(1 - \frac{K(0)}{K^*} \right)^2}{(1 + M)^2} (> 0), \quad (52)$$

which is analytically evident to see that the long-run level of wealth inequality is greater than the initial level, $S^* > 0 (= S(0))$.

Finally, let us consider the relationship between the wealth inequality and the speeds of convergence from the viewpoint of heterogeneous conformism. As seen in (27) and (29), the speeds of convergence become faster if $\int_0^1 \Lambda_i(c_i^*, C^*) di$ approaches to the unity. When we make use of (30), such an economy means that the long-run relative-wealth rich has a greater degree of conformism. Turning our interests into Proposition 4, from (24) we can see that the dispersion of the long-run elasticity of intertemporal substitution in (#3) becomes larger, which means that the wealth inequality tends to expand. Furthermore, from (#4) if the long-run relative-wealth rich is rich in the initial period as well, the dispersion of wealth becomes larger.

4.3 Numerical Analysis

Finally, we examine numerical examples that may capture the central message of our study in the simplest manner. To do so, we assume that there are only two types of households. The instantaneous utility of each type of household is (20) :

$$u(c_i, C) = \frac{(c_i C^{-\theta_i})^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad \gamma \neq 1, \quad 0 < \theta_i < 1, i = 1, 2.$$

It is assumed that type 1 households constitute a continuum with a mass of $\beta \in (0, 1)$. They have identical initial capital, $k_1(0)$. The second group of households is also a continuum with a mass of $1 - \beta$ whose initial capital is $k_2(0)$. By definition, the aggregate (average) consumption is $C = \beta c_1 + (1 - \beta) c_2$. In what follows, we assume that type 1 households are initially richer than type 2 households, and thus $k_1(0) > k_2(0)$.

The production function is given by Cobb-Douglas: $Y = AK^\alpha$ where $0 < A$ and $0 < \alpha < 1$. Therefore, irrespective of the values of production parameters A and α , the condition (37) is always satisfied, which means that if $\theta_1 = \theta_2$ so that both types of households have an identical degree of conformism, then the level of wealth inequality in the long run is less than

the initial level, that is, $S^* < S(0)$. In the following, we set:

$$A = 1, \quad \alpha = 0.35, \quad \gamma = 2.5, \quad \rho = 0.04.$$

From (26b), the interest rate is 0.04 and the long-run level of aggregate capital is given by $K^* = 19.96$. Furthermore, we assume the initial level of aggregate capital equals 80 % level of its steady-state one, that is, $K(0) = 0.8K^*$, which implies that our economy is growing over time.

As to the population size and the initial distribution of capital, we consider the following four cases:

- (i) $\beta = 0.5, \quad \sigma_1(0) = 0.1,$
- (ii) $\beta = 0.5, \quad \sigma_1(0) = 0.01,$
- (iii) $\beta = 0.2, \quad \sigma_1(0) = 0.1$
- (iv) $\beta = 0.2, \quad \sigma_1(0) = 0.01.$

Cases (i) and (ii) assume that the population size of each group is the same. Cases (iii) and (iv) consider a more realistic situation where the rich group (type 1) has a smaller size of population than the poor one (type 2). As for the initial distribution of capital, Cases (i) and (iii) assume that the initial capital stock held by type 1 households is 10% higher than the average. Since it holds that $\beta\sigma_1(0) + (1 - \beta)\sigma_2(0) = 0$, the initial capital holding of type 2 households is $\sigma_2(0) = -0.1$ in Case (i) and $\sigma_2(0) = -0.025$ in Case (iii). Similarly, the levels of relative wealth in Cases (ii) and (iv), $\sigma_1(0) = 0.01$ are smaller than in Cases (i) and (iii) where $\sigma_2(0) = -0.01$ in Case (iii) and $\sigma_2(0) = -0.0025$ in Case (iv), meaning that the differences of wealth between households in rich and poor groups are smaller in Cases (ii) and (iv).

Given those parameter magnitudes, we change θ_1 and θ_2 in the range that $0 \leq \theta_i \leq 1$. All figures divide (θ_1, θ_2) space according to whether the level of wealth inequality in the steady state is larger than its initial one.¹⁸ The areas with a red triangle show the combination of

¹⁸We derive the steady-state levels of individual capital as follows. First, using (20) and $C^* = \beta c_1^* + (1 - \beta)c_2^*$, the ratio of marginal utility $u_1(c_1^*, C^*)/u_1(c_2^*, C^*) = \epsilon$ leads to $c_1^* = c_1^*(\epsilon)$ where $\epsilon(> 0)$ is an unknown constant parameter, leading to $k_1^* = k_1^*(\epsilon)$ under (28), and hence, $\sigma_1^* = \sigma_1^*(\epsilon)$. Substituting $k_1^* = k_1^*(\epsilon)$ and $\sigma_1^* = \sigma_1^*(\epsilon)$ into (44a) at $t = 0$, we can obtain the unique relationship between ϵ and $k_1(0)$ where we make use of $\beta\sigma_1(0) + (1 - \beta)\sigma_2(0) = 0$. As a result, we can obtain $k_1^* = k_1^*(\epsilon(k_1(0)))$.

θ_1 and θ_2 that yields the expansion of wealth inequality in the long run. Alternatively, the areas with a black cross mean that the wealth inequality shrinks in the long run.

Figure 1(a) depicts Case (i). Here, we see that the lower-right area is occupied by the signs of red triangle. In other words, when the value of θ_1 is somewhat greater than that of θ_2 , the long-run level of wealth inequality is greater than its initial one, $S^* > S(0)$, meaning that when the households in rich group have the stronger degrees of conformism than those in poor group, the wealth inequality tends to expand. However, if the difference between θ_1 and θ_2 is small enough, the wealth inequality between both types of households will reduce in the long run. This is because the right-hand side of (47) has the larger value than the left-hand side. In other words, the initial-rich households achieve the larger jump of private consumption at the initial period, and therefore, the difference in wealth between two types of households will be lowered. When the difference in the degree of conformism between the two groups is sufficiently small, the effect of initial jump of private consumption is kept over time. As a result, the long-run level of wealth inequality decreases. In contrast, when the households in poor group have the greater degrees of conformism than the others, the wealth inequality is reduced. Furthermore, it is to be noted that when the degrees of conformism in both groups are the same ($\theta_1 = \theta_2$), the wealth inequality shrinks as seen in the last subsection. This result reconfirms García-Peñalosa and Turnovsky's (2008) main finding in our context.

Figure 1(b) corresponds to Case (ii). There are two differences from Figure 1(a). First, if the degree of conformism of initial rich is slightly larger than that of the initial poor, the wealth inequality may expand in the long run. The reason is that in the Case (ii) the initial jump of private consumption of the rich group is slightly larger than that in poor group because the initial levels of capital in both groups trivially differ. Consequently, the heterogeneity of conformism has a larger impact on the wealth inequality so that the dispersion of wealth becomes larger in the long run. Second, and more interestingly, the upper-left area is occupied by the signs of red triangle, which implies that when the degrees of conformism in poor group are larger than those in rich group to some extent, the level of wealth inequality in the steady state is greater than that at the initial period. In other words, the initially poorer households catch up with the households who are initially rich as seen in Proposition 2, and after the reversal of wealth arises, the difference of wealth expands over time. As a result, the long-run level of wealth inequality is larger than its initial level.

Figures 2(a) and 2(b) respectively depict Cases (iii) and (iv) where the population size of the initial rich (type 1) is 0.2. The patterns of long-run wealth distribution in the case of $\beta = 0.2$ are similar to those observed in Figures 1(a) and 1(b) in which $\beta = 0.5$. However, there are quantitative differences between the two cases: the areas which show the expansion of wealth inequality are larger in Figures 2(a) and 2(b) than those in Figures 1(a) and 1(b). This implies that even if the difference of degrees of conformism is small, the wealth inequality may expand if there is a substantial gap in the population sizes of two groups. It is interesting to note that as the population size of households who initially rich becomes smaller, the long-run wealth inequality will expand.¹⁹

5 Conclusion

This paper introduces preference heterogeneity into García-Peñalosa and Turnovsky (2008). We have studied how the presence of heterogeneous degrees of conformism among households affects the aggregate dynamics as well as wealth distribution in the long run. We have presented three main results. First, Proposition 1 of the paper shows that an economy where households have stronger conformism on average grows faster. This result confirms that the finding in the representative-agent models with consumption externalities still holds even if households have heterogeneous preferences.

Second, it is shown that if households' preferences are heterogeneous, an initially poor household may catch up with initially rich households, as long as her consumption conformism is strong enough. The precise conditions for such a catch up are given in Proposition 2. This proposition also demonstrates that the catch up will not arise if households' preference are identical and homothetic with respect to private and the average consumption.

Third, it is revealed that because of the presence of heterogeneous conformism, the wealth inequality may be enhanced in the long run. This is in contrast to the case of homogeneous and homothetic preferences under which wealth inequality tend to be reduced during the transition. Propositions 3 and 4 present a set of explicit conditions that determine the long-

¹⁹We have seen that the speeds of convergence become faster relative to the economy in which households have no conformism. In other words, even if a portion of households has degrees of conformism but the rest households have no conformism, the economy which has a degree of conformism on average has the faster speed of convergence. This finding is likely to support Proposition 1(i).

run pattern of wealth inequality. Finally, our numerical examples show the possibilities of catch-up by the initially less wealth person, and the expansion of wealth inequality under the plausible parameter set.

Using our model, we can re-examine the existing studies on consumption externalities with heterogeneous agents. For example, Koyuncu and Turnovsky (2010) analyze the role of tax policy in the context of García-Peñalosa and Turnovsky's (2008) model. Our model with heterogeneous preference may extend their policy analysis. In addition, Gollier (2004) studies a static, general equilibrium model of asset market with consumption externalities and heterogeneous preferences. Based on our framework, we may present a dynamic version of Gollier's (2004) discussion. Those topics would deserve further study.

Appendices

Appendix A

We derive the equation (35) and (39) where the derivation is fundamentally the same as García-Peñalosa and Turnovsky (2006, 2008). First, substituting the individual as well as the aggregate capital accumulation equations into $\dot{\tilde{k}}_i = \dot{k}_i/K - \tilde{k}_i\dot{K}/K$ and arranging for it, we can show

$$\dot{\tilde{k}}_i = \frac{1}{K} \left\{ (f'(K)K - f(K))(\tilde{k}_i - 1) + C(\tilde{k}_i - \frac{c_i}{C}) \right\}. \quad (\text{A.1})$$

Identical preferences: Note that the relative consumption $\frac{c_i}{C}$ is constant over time under the identical preferences. Approximating (A.1) around the steady state, we can obtain

$$\dot{\tilde{k}}_i = \rho(\tilde{k}_i - \tilde{k}_i^*) + f''(K^*)(\tilde{k}_i^* - 1)(K - K^*) + \frac{\tilde{k}_i^* - c_i^*/C^*}{K^*}(C - C^*), \quad (\text{A.2})$$

and finally arranging for it, we can derive (35).

Heterogeneous preferences: Since the relative consumption $\frac{c_i}{C}$ is not constant, the linear approximation (A.1) around the steady state is

$$\dot{\tilde{k}}_i = \rho(\tilde{k}_i - \tilde{k}_i^*) + f''(K^*)(\tilde{k}_i^* - 1)(K - K^*) + \frac{1}{K^*} \left\{ \tilde{k}_i^*(C - C^*) - (c_i - c_i^*) \right\}. \quad (\text{A.3})$$

Therefore, using $C - C^* = (\rho - \mu_s)(K - K^*)$ and (33), we can show (39) where we use $1 = \frac{\Delta^* f''(K^*)}{\mu_s(\rho - \mu_s)}$ derived by summing (32) over all households.

Appendix B

Raising both sides of (44a) to the double power at $t = 0$ and summing up for all households yields:

$$S(0) = S^* + \frac{2(K^* - K(0))}{\rho - \mu_s} \int_0^1 \sigma_i^* Z_i^* di + \left(\frac{K^* - K(0)}{\rho - \mu_s} \right)^2 \int_0^1 (Z_i^*)^2 di, \quad (\text{B.1})$$

where

$$\begin{aligned} \int_0^1 Z_i^* \sigma_i^* di &= \frac{B^* S^*}{K^*} - \frac{\rho - \mu_s}{\Delta^* K^*} \int_0^1 \Phi_i^* \sigma_i^* di, \\ \int_0^1 (Z_i^*)^2 di &= \frac{(B^*)^2 S^*}{(K^*)^2} - \frac{2B^*(\rho - \mu_s)}{\Delta^* (K^*)^2} \int_0^1 \Phi_i^* \sigma_i^* di + \left(\frac{\rho - \mu_s}{K^*} \right)^2 \left(-1 + \frac{\int_0^1 (\Phi_i^*)^2 di}{\left(\int_0^1 \Phi_i^* \right)^2} \right). \end{aligned}$$

As a consequence, we can show

$$S(0) = (1 + M^*)^2 S^* - D^*, \quad (\text{B.2a})$$

where M^* is defined by (48) and

$$D^* = \left(1 - \frac{K(0)}{K^*}\right)^2 \left(1 - \frac{\int_0^1 (\Phi_i^*)^2 di}{(\int_0^1 \Phi_i^* di)^2}\right) + 2(1 + M^*) \left(1 - \frac{K(0)}{K^*}\right) \frac{\int_0^1 \sigma_i^* \Phi_i^* di}{\int_0^1 \Phi_i^* di}. \quad (\text{B.2b})$$

Notice that $\Delta^* = \int_0^1 \Phi_i^* di$.

To derive (50), we can rewrite (B.2b). We make use of (44a) at the initial time and rewrite the equation as follows:

$$\sigma_i^* = \frac{1}{1 + M^*} \left(\sigma_i(0) + \left(1 - \frac{K(0)}{K^*}\right) \left(\frac{\Phi_i^*}{\int_0^1 \Phi_i^* di} - 1 \right) \right), \quad (\text{B.3a})$$

and furthermore we can obtain the following:

$$\int_0^1 \sigma_i^* \Phi_i^* di = \frac{1}{1 + M^*} \left(\int_0^1 \sigma_i(0) \Phi_i^* di + \left(1 - \frac{K(0)}{K^*}\right) \left(\frac{\int_0^1 (\Phi_i^*)^2 di}{(\int_0^1 \Phi_i^* di)^2} - 1 \right) \int_0^1 \Phi_i^* di \right). \quad (\text{B.3b})$$

Finally, substituting (B.3b) into (B.2b), D^* corresponds to X^* in (50).

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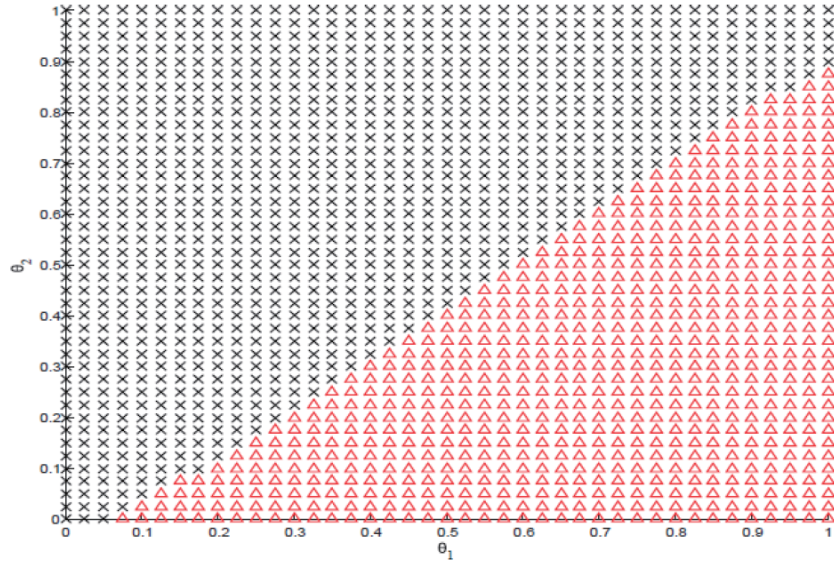


Figure 1(a): $\beta = 0.5$ and $\sigma_1(0) = 0.1$

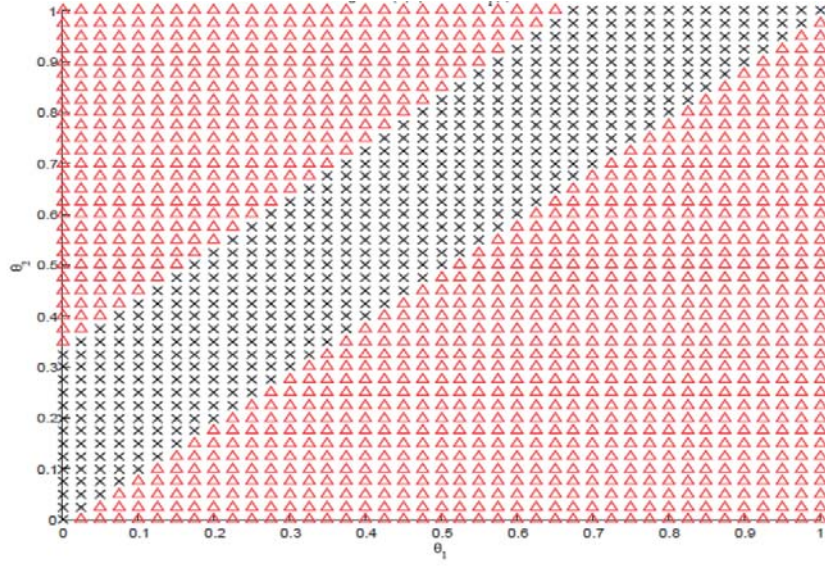


Figure 1(b): $\beta = 0.5$ and $\sigma_1(0) = 0.01$

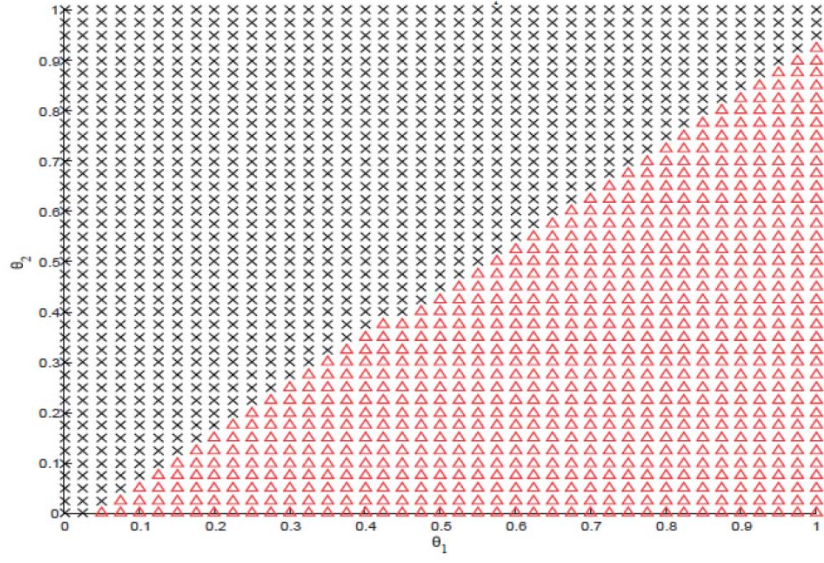


Figure 2(a): $\beta = 0.2$ and $\sigma_1(0) = 0.1$

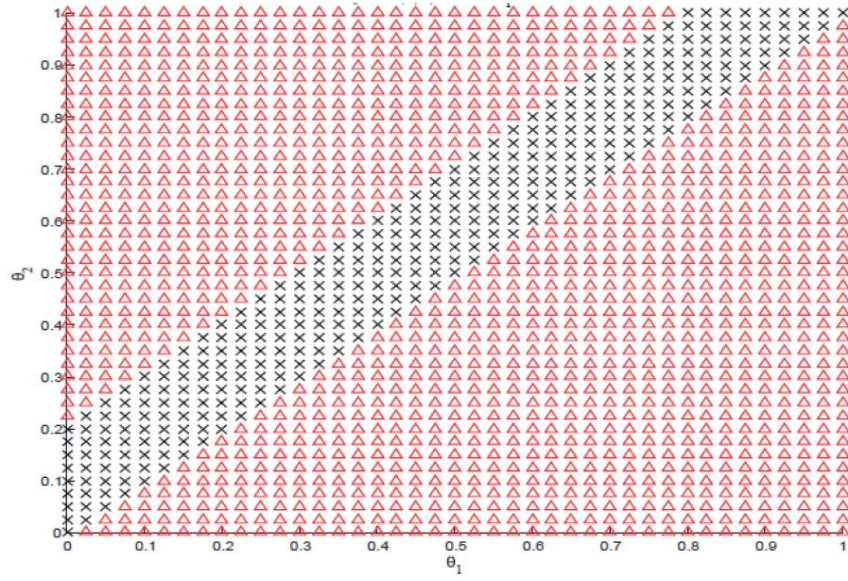


Figure 2(b): $\beta = 0.2$ and $\sigma_1(0) = 0.01$