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Small-Open Economy**

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Capital Accumulation and Structural Change in a Small-Open Economy*

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Abstract

This paper explores the relation between capital accumulation and transformation of industrial structure in a small open-economy. Using a three-sector, neoclassical growth model with non-homothetic preferences, we examine dynamic behavior of the small country in the alternative trade regimes. We show that capital accumulation plays a leading role in the process of structural transformation. It is also revealed that the trade pattern significantly affects structural change. We demonstrate that our model can mimic a typical pattern of change in industrial structure that has been observed in many developed economies.

Keywords: Structural change, Small-open economy, Trade Pattern, Three-sector model, Non-homothetic preferences

JEL classification numbers: E21, O10, O41

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1 Introduction

Recently, there has been a renewed interest in the relation between economic growth and transformation of industrial structure. Although the recent investigations have presented more sophisticated analytical frameworks than those used by the earlier literature in the 1960s, they cannot fully capture cross-country differences in structural change: see Buera and Kaboski (2009) for a critical evaluation of recent research outcomes. One of the reasons for the presence of gap between model predictions and empirical observations may stem from the fact that most of the recent investigations on growth and structural change have employed closed economy models. As is well recognized, the dynamic behavior of an open economy would be substantially different from that of the closed economy counterpart. In addition, as Matsuyama (2009) emphasizes, there is no closed economy in our real world and the only closed economy we know is the global economy itself. Therefore, it is a relevant task to reconsider structural change in open-economy settings.

According to such a research agenda, we examine structural transformation of a growing open economy. The central concern of this paper is to show that the standard neoclassical growth model with non-homothetic preferences may exhibit the empirically plausible relation between economic growth and structural change. We use a three-sector, neoclassical growth model with non-homothetic preferences. The analytical framework of our discussion is an open-economy version of Kongsamut et al. (2001). Unlike Kongsamut et al. (2001), we assume that each production sector selects a different level of capital intensity, which enable us to empathize the role of capital accumulation in the process of structural change.¹ We assume that one sector produces manufacturing goods that can be used either for consumption or for investment. Other two sectors produce two kinds of pure consumption goods: agricultural goods and services. It is also assumed that the manufacturing and agricultural goods are internationally traded, but services are consumed in the domestic market alone. Due to the presence of non-traded goods, the pattern of trade depends not only on production technologies but also on the consumption demand for nontraded goods.

Given the baseline setting mentioned above, we consider a small-open economy where the

¹Kongsamut et al. (2001) assume that each production sector uses the same form of constant-return-to scale technology, meaning that each sector chooses the same capital-labor ratio. Due to this restriction, the aggregated model behaves like a one-sector growth model.

terms of trade between tradable goods is determined in the rest of the world. It is assumed that at the outset the rest of the world has already reached the steady state and that the initial stock of aggregate capital held by the small country is much smaller than the steady-state level of capital in the rest of the world. We first characterize the steady-state equilibrium of the closed economy and then explore the behavior of the small-open economy.

Our main results are as follows. First, if the small country has the same technologies and preferences as those of the rest of the world, the small country and the rest of the world have the same steady state. In this case, the small country specializes in the agricultural goods during the early state of development. As capital accumulates, the small country starts producing the manufacturing goods as well. Then during the transition towards the steady state, both the manufacturing good and service sectors expand and the relative income share of agricultural sector decreases. On the other hand, if there is asymmetry in technologies and/or preferences between the small country and the rest of the world, then the small country may have a steady state in the regime specializing in the manufacturing goods, In this regime, the small country does not produce the agricultural goods. Here, the relative income share of the manufacturing sector may decline, while the share of the service sector continues increasing. Combining the non-specialization and specialization regimes, we see that the income share of the agricultural sector continues falling and that of the service sector increases. The income share of the manufacturing good sector first rises and then starts declining as the economy moves into the specialization regime. Such a pattern of change in industrial structure can capture a typical structural change generally observed in developed countries.

In the recent literature, several authors examine structural change in open economy settings. Atkeson and Kehoe (2000) study a dynamic Heckscher-Ohlin model and consider the effect of trade pattern on capital formation of a small country. Since they use the standard two sector model in which one sector produces pure consumption goods and the other produces pure investment goods, structural change in the usual sense is not fully discussed. Uy, Yi and Zhang (2013) construct a two-country, three-sector model of the world economy. While their world economy setting is more general than ours, they employ a Ricardian model where production of each good needs labor alone, so that the role of capital accumulation is not discussed. Using a simple two-country, three sector model, Matsuyama (2009) also examines the impacts of international trade on structural change. As well as Uy, Yi and Zhang (2013), Matsuyama (2009) does not consider

capital accumulation.² Teigniere (2012) examines structural change in a small-open economy with capital accumulation, but the author uses a two-sector model so that a hump-shaped profile of manufacturing cannot be discussed. Mao and Yao (2012) explore a three-sector open economy model in which one sector produces nontraded goods and the other two produce tradable goods. Thus the analytical framework of their study is close to ours. The main difference is that while Mao and Yao (2012) emphasize the unbalanced productivity growth between the production sectors, our paper focuses on the role of trade pattern. Since both productivity change and trade play relevant roles in the process of structural transformation in open economies, Mao and Yao (2012) and our study are complements rather than substitutes.

The rest of the paper is organized as follows. Next section displays the baseline setting. Section 3 characterizes the steady-state equilibrium of the closed economy. Section 4 analyzes the patterns of trade of the small-open economy and examines its dynamic equilibrium paths under alternative trade patterns. Section 5 discusses the patterns of structural change under three different regimes. Section 6 concludes.

2 Base Model

The analytical framework of our discussion is basically the same as that of Kongsamut et al. (2001). Our departure from their model is that we assume that each production sector selects a different factor intensity. Kongsamut et al. (2001) assume that all industries chose the same factor intensity, so that the relative price depends only on the relative total factor productivity.

2.1 Production

There are three production sectors: manufacturing, agricultural and service sectors. Sector m produces manufacturing goods that can be either consumed or invested for capital formation. Sectors a and s respectively produce agricultural goods and services both of which are pure consumption goods.³ When discussing open economies, we assume that the manufacturing and

²An earlier contribution to structural change in an open economy was presented by Matsuyama (1992) who considers a two-sector model with leaning-by-doing. Dei (1998) also examines industrialization of a small-open economy with nontraded intermediate goods and shows that the small country may have multiple equilibria. These authors do not consider capital accumulation either.

³Earlier studies on the three-sector model with a non-traded good are Ethier (1972) and Komiyama (1967).

agricultural good are internationally traded, but services are nontradables. Each production sector employs capital and labor under a constant-returns-to-scale technology. The production function of each sector is specified as

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i} = L_i A_i k_i^{\alpha_i}, \quad 0 < \alpha_i < 1, \quad i = a, m, s.$$

where Y_i , K_i and L_i respectively denote output, capital and labor in the i -th sector. In addition, A_i denotes the total factor productivity of sector i and $k_i (= K_i/L_i)$ is the capital-labor ratio in that sector.

Factor and goods markets are competitive. Thus profit maximization of producers yields the following conditions:

$$r = \alpha_m \frac{Y_m}{K_m} = p_a \alpha_a \frac{Y_a}{K_a} = p_s \alpha_s \frac{Y_s}{K_s}, \quad (1)$$

$$w = (1 - \alpha_m) \frac{Y_m}{L_m} = p_a (1 - \alpha_a) \frac{Y_a}{L_a} = p_s (1 - \alpha_s) \frac{Y_s}{L_s}. \quad (2)$$

where r is the rate of return to capital and w is the real wage rate. In addition, p_a and p_s respectively denote the prices of agricultural good and services in terms of the manufacturing good,⁴

Conditions (1) and (2) show that the marginal rate of substitution in every sector has the same magnitude:

$$\frac{(1 - \alpha_i) k_i}{\alpha_i} = \omega, \quad i = a, m, s,$$

where ω expresses the factor price ratio, w/r . The above equation gives the relation between the capital intensity and the factor price ratio in such a way that

$$k_i = \beta_i \omega, \quad \beta_i = \frac{\alpha_i}{1 - \alpha_i}, \quad a, m, s. \quad (3)$$

From (1) and (2) we also obtain the following:

⁴Kongsamut et al. (2001) assume that each production function is given by $Y_i = A_i F(K_i, L_i)$ ($i = a, m, s$), so that each sector selects the same capital intensity. Thus the relative price depends on the TFPs alone: $p_a = A_m/A_a$ and $p_s = A_m/A_s$. As a result, the aggregate income (in terms of the first good) is written as $Y = Y_m + p_a Y_a + p_s Y_s = A_1 F(K, L)$, because it holds that $k_i = k = K/L$ for $i = a, m, s$. Such a simple aggregation is not possible in our setting.

$$p_i = \frac{\alpha_m A_m \beta_m^{\alpha_m - 1}}{\alpha_i A_i \beta_i^{\alpha_i - 1}} (\omega)^{\alpha_m - \alpha_i}, \quad i = a, s. \quad (4)$$

Hence, from (4), we may express the relation between p_a , p_s and ω in the following manner:

$$p_a = p^a(\omega; A_m/A_a), \quad p_s = p^s(\omega, A_m/A_s).$$

In most of the subsequent analysis, we express $p_a = p_a(\omega)$ and $p_s = p_s(\omega)$ for simplicity. We see that

$$\text{sign } p'_i(\omega) = \text{sign}(\alpha_m - \alpha_i) = \text{sign}(\beta_m - \beta_i), \quad i = a, s. \quad (5)$$

In what follows, we assume that the manufacturing good sector always selects the most capital intensive technology, while the service sector uses the most labor intensive technology. That is, we assume:

$$\alpha_m > \alpha_a > \alpha_s, \quad (6)$$

so that $\beta_m > \beta_a > \beta_s$. Consequently, the relation between ω , p_a and p_s are assumed to be

$$p'_i(\omega) > 0, \quad i = a, s. \quad (7)$$

As discussed below, when we treat an open economy, we assume that production factors will not cross the borders. Thus in both closed and open economies, the full-employment conditions for capital and labor are:

$$K_a + K_m + K_s = K,$$

$$L_a + L_m + L_s = 1,$$

where K and L the aggregate levels of capital and labor, respectively. We assume that the total labor supply L is constant and normalized to one. Note that the full-employment of capital is rewritten as

$$k_a L_a + k_m L_m + k_s L_s = k. \quad (8)$$

where $k = K/L (= K)$ is the capital intensity of the economy at large. From (3) the above

expression leads to

$$\beta_a L_a + \beta_m L_m + \beta_s L_s = \frac{k}{\omega}. \quad (9)$$

2.2 Consumption

There is a continuum of infinitely lived households with a unit mass. Each household supplies one unit of labor in each moment. Following Kongsamut et al. (2001), we assume that the instantaneous utility function of the household is given by the following Stone-Geary function:

$$\begin{aligned} u(c_a, c_m, c_s) &= c_m^{\gamma_m} (c_a - \bar{c}_a)^{\gamma_a} (c_s + \bar{c}_s)^{\gamma_s}, \\ \bar{c}_i &> 0, \quad \gamma_i > 0, \quad \gamma_a + \gamma_m + \gamma_s = 1., \quad i = a, m, s, \end{aligned}$$

where c_i ($i = q, m, s$) denote consumption level of good i . We first solve the household's instantaneous optimization problem such that

$$\max u(c_a, c_m, c_s)$$

subject to $c_m + p_a c_a + p_s c_s = E$, where E denotes the instantaneous income. Solving this problem gives the relations between the optimal levels of c_m , c_a and c_s in the following manner:

$$c_a = \frac{\gamma_a}{p_a \gamma_m} c_m + \bar{c}_a, \quad (10)$$

$$c_s = \frac{\gamma_s}{p_s \gamma_m} c_m - \bar{c}_s. \quad (11)$$

Substituting these values into the utility function, we rewrite $u(\cdot)$ in such a way that

$$\hat{u}(c_m, p_a, p_s) = \tilde{B}(p_a, p_s) c_m,$$

where

$$\tilde{B}(p_a, p_s) = \left(\frac{\gamma_a}{p_a \gamma_m} \right)^{\gamma_a} \left(\frac{\gamma_s}{p_s \gamma_m} \right)^{\gamma_s}.$$

The dynamic optimization problem for the household is to maximize

$$U = \int_0^\infty e^{-z t} \frac{1}{1-\sigma} \tilde{B}(p_a, p_s)^{1-\sigma} c_m^{1-\sigma} dt \quad (12)$$

subject to

$$\dot{k} = rk + w - c_m - p_a c_a - p_s c_s,$$

as well as to the initial capital holding, k_0 . In this paper we assume that the time discount rate of the household is an endogenous variable whose behavior is given by

$$\dot{z} = \rho(\hat{c}_m), \quad \rho'(\hat{c}_m) > 0,$$

where \hat{c}_m denotes the average consumption level of the manufacturing good. We assume that $\rho(\hat{c}_m)$ is an increasing function of \hat{c}_m . This formulation is similar to the Koopmans-Usawa modelling of the endogenous time preference with increasing marginal impatience. Here, it is assumed that the time preference of each household does not depend on its private consumption but on the social level of consumption, which is represented by the average consumption of the manufacturing goods in the economy at large. The main rationalization for our setting is that other households' consumption levels of agricultural goods and services are harder to observe than the manufacturing goods such as cloths, electrical appliances, cars and houses.⁵ In addition, if the time discount rate depends on the total consumption expenditure, $\rho(\cdot)$ involves the relative prices, p_a and p_s , which would add an unnecessary complexity to our model manipulation.⁶

In our setting, when selecting the optimal consumption plan, the households takes the sequence of external effects, $\{\hat{c}_m(t)\}_{t=0}^{\infty}$ as given. In equilibrium it holds that $\hat{c}_m = c_m$. Thus letting λ be the implicit price of capital, the optimization conditions give

$$\frac{1}{1-\sigma} \tilde{B}(p_a, p_s)^{1-\sigma} c_m^{-\sigma} = \lambda e^z, \quad (13)$$

$$\dot{\lambda} = -r\lambda, \quad (14)$$

together with the transversality condition: $\lim_{t \rightarrow \infty} e^{-z} \lambda k = 0$. Note that in equilibrium where

⁵For example, based on survey data analyses, Alpizar *et al.* (2005), Solnick and Hemenway (2005), Carlsson *et al.* (2007) conclude that goods with high observability, such as cars and houses, have strong external effects.

⁶The total consumption expenditure is $c_m + p_a c_a + p_s c_s = [(\gamma_a + \gamma_s) / \gamma_m] c_m + \bar{c}_a p_a - \bar{c}_s p_s$. In a closed economy, it turns out that the equilibrium levels of p_a and p_s are functions of c_m and k , so that ρ is a function of k as well as c_m if ρ is determined by the average level of the total consumption spending. Although it is possible to conduct model analysis in this generalized condition, we need additional restrictions of the parameter magnitude to obtain clear conclusions.

$\hat{c}_m = c_m$, z changes according to

$$\dot{z} = \rho(c_m). \quad (15)$$

3 The Rest of the World

As assumed in Atkeson and Kehoe (2000), we focus on the behavior of a small-open economy when the rest of the world has already reached the steady state. The rest of world is assumed to consists of a continuum of identical countries and they are fully integrated. Thus the rest of the world behaves like a closed economy. Therefore, we first characterize the steady state equilibrium of the closed economy.⁷

3.1 Market Equilibrium and Dynamic System

Since the rest of the world is assumed to be a closed economy, the market clearing conditions for the agricultural and manufacturing goods are $Y_i = c_i$ ($i = a, s$), so that

$$Y_a = \frac{\gamma_a}{p_a \gamma_m} c_m + \bar{c}_a \quad (16)$$

$$Y_s = \frac{\gamma_s}{p_s \gamma_m} c_m - \bar{c}_s. \quad (17)$$

The equilibrium condition for the manufacturing good market is:

$$Y_m = \dot{k} + c_m. \quad (18)$$

For simplicity, we ignore capital depreciation.

⁷The closed economy version of our model is also related to Acemoglu and Guerriel (2008), Foellmi and Zweimuller (2008), Hori et al. (2013), Iscan (2010). Laitner (2000) and Nagi and Pissaides (2007). As well as in our model, Acemoglu and Guerriel (2008) emphasizes the role of factor intensity differences between the production sectors in the context of two-sector, closed economy model. Nagi and Pissarides (2007), on the other hand, focus on the changes in productivity gap between production sectors emphasized by Baumol (1967). The role of non-homothetic preferences is discussed in various settings by Foellmi and Zweimuller (2008), Hori et al. (2013), Laitner (2000) and Iscan (2010).

Using (8) and $L_a + L_m + L_s = 1$, we obtain:

$$\begin{aligned} L_m &= \frac{k - k_a + (k_a - k_s) L_s}{k_m - k_a} \\ &= \frac{k - \beta_a \omega + (\beta_a - \beta_s) \omega L_s}{(\beta_m - \beta_a) \omega}, \end{aligned} \quad (19)$$

$$\begin{aligned} L_a &= \frac{k_m - k - (k_m - k_s) L_s}{k_m - k_a} \\ &= \frac{\beta_m \omega - k - (\beta_m - \beta_s) \omega L_s}{(\beta_m - \beta_a) \omega}. \end{aligned} \quad (20)$$

Equation (17) and $Y_s = c_s$ present

$$L_s = \frac{c_s}{A_s k_s^{\alpha_s}} = \frac{1}{A_s (\beta_s \omega)^{\alpha_s}} \left[\frac{\gamma_s}{p_s(\omega) \gamma_m} c_m - \bar{c}_s \right] = L^s(\omega, c_m). \quad (21)$$

We see that

$$L_\omega^s(\omega, c_m) < 0, \quad L_{c_m}^s(\omega, c_m) > 0.$$

Moreover, (16) is written as

$$\begin{aligned} Y_a &= L_a A_a k_a^{\alpha_a} \\ &= \frac{\beta_m \omega - k - (\beta_m - \beta_s) \omega L^s(\omega, c_m)}{(\beta_m - \beta_s) \omega} A_a (\beta_a \omega)^{\alpha_a} \\ &= \frac{\gamma_a}{p_a(\omega) \gamma_m} c_m + \bar{c}_a. \end{aligned} \quad (22)$$

This equation relates the factor price ratio, ω , to k and c_m in such a way that

$$\omega = \Omega(k, c_m).$$

Substituting this into $Y_m = L_m A_m k_m^{\alpha_m}$ yields

$$\begin{aligned} Y_m &= L_m A_m k_m^{\alpha_m} = \frac{k - \beta_a \omega + (\beta_a - \beta_s) \omega L^s(\omega, c_m)}{(\beta_m - \beta_a) \omega} A_m (\beta_m \omega)^{\alpha_m} \\ &= y^m(k, \Omega(k, c_m)) \end{aligned} \quad (23)$$

Finally, the optimization condition (13) leads to

$$B (p_a(\Omega(k, c_m)), p_s(\Omega(k, c_m)))^{1-\sigma} c_m^{-\sigma} = \lambda e^z,$$

implying that

$$(1 - \sigma) \frac{\dot{B}}{B} - \sigma \frac{\dot{c}_m}{c_m} = \rho(c_m) - r$$

Consequently, a complete dynamic system of the closed economy consists of the following set of differential equations:

$$\dot{k} = y^m(k, \Omega(k, c_m)) - c_m, \quad (24)$$

$$\dot{c}_m = \frac{c_m}{\sigma} \left[\alpha_m A_m (\beta_m \Omega(k, c_m))^{\alpha_m - 1} - \rho(c_m) + (1 - \sigma) \frac{\dot{B}}{B} \right], \quad (25)$$

where

$$B = \left(\frac{\gamma_a}{p_a(\Omega(k, c_m)) \gamma_m} \right)^{\gamma_a} \left(\frac{\gamma_s}{p_s(\Omega(k, c_m)) \gamma_m} \right)^{\gamma_m}$$

3.2 The Steady-State Equilibrium

Although the functional forms involved in (24) and (25) are rather complex, the steady-state characterization of the closed economy is simple one. It is easy to confirm that in the steady state where k and c_m stay constant, the following conditions are satisfied:

$$\rho(c_m) = \alpha_m A_m (\beta_m \Omega(k, c_m))^{\alpha_m - 1}, \quad (26)$$

$$y^m(k, \Omega(k, c_m)) = c_m.$$

These two equations may determine the steady-state levels of k and c_m , so is the steady state value of ω . Once the steady-state values of k and c_m are determined, the steady-state levels of labor allocation, L_i ($i = a, m, s$), the relative prices, p_a and p_s as well as the factor prices, r and w are uniquely given, because all of them are functions of c_m and/or ω . In the following discussion, we assume that the rest of the world has a unique steady state.

4 Behavior of a Small-Open Economy

As was mentioned, we assume that the integrated rest-of-the-world has already reached the steady state where each variable satisfies conditions discussed in the previous section. We focus on the dynamics and steady-state equilibrium of a small country.

4.1 Non-Specialization

We first consider the situation where the small country does not specialize, so that she produces both agricultural and manufacturing goods along with services. Since the rest of the world stays in the steady-state equilibrium, the terms of trade between the agricultural and manufacturing goods is the steady-state price level of p_a held in the rest of the world. We express the world level of p_a as \hat{p}_a . Given \hat{p}_a , the factor price ratio in the small county is determined by

$$\hat{p}_a = \frac{\alpha_m A_m \beta_m^{\alpha_m - 1}}{\alpha_a A_a \beta_a^{\alpha_a - 1}} (\omega^*)^{\alpha_m - \alpha_a}, \quad (27)$$

where ω^* is the factor price ratio in the small country when she produces both manufacturing and agricultural goods under free trade. As a consequence, the relative price of non-traded services, p_s , is also fixed by the following:⁸

$$p_s^* = \frac{\alpha_m A_m \beta_m^{\alpha_m - 1}}{\alpha_s A_s \beta_s^{\alpha_s - 1}} (\omega^*)^{\alpha_m - \alpha_s}. \quad (28)$$

Remember that when the small country produces both agricultural and manufacturing goods, the labor allocation to each sector is respectively given by

$$L_m = \frac{k - \beta_a \omega^* + (\beta_a - \beta_s) \omega^* L^s(\omega^*, c_m)}{(\beta_m - \beta_a) \omega^*},$$

$$L_a = \frac{\beta_m \omega^* - k - (\beta_m - \beta_s) \omega^* L^s(\omega^*, c_m)}{(\beta_m - \beta_a) \omega^*}.$$

where

$$L^s(\omega^*, c_m) = \frac{1}{A_s (\beta_s \omega^*)^{\alpha_s}} \left[\frac{\gamma_s}{p_s(\omega^*) \gamma_m} c_m - \bar{c}_s \right].$$

⁸When there is no difference in technologies and preferences between the home and foreign countries, p_s^* also stands for the steady state level of p_s established in the rest of the world.

Thus to keep the conditions $L_a > 0$ and $L_m > 0$, the levels of c_m and k should satisfy the following:

$$k + (\beta_a - \beta_s) \omega^* L^s(\omega^*, c_m) > \beta_a \omega^*, \quad (29)$$

$$k + (\beta_m - \beta_s) \omega^* L^s(\omega^*, c_m) < \beta_m \omega^*. \quad (30)$$

We assume that the representative household in the small country has the same preference as the rest of the world and, hence, the household maximizes U in (12) subject to

$$\dot{k} = rk + w - c_m - \hat{p}_a c_a - p_s^* c_s. \quad (31)$$

The optimization conditions yield:

$$B(p_a(\omega^*), p_s(\omega^*))^{1-\sigma} c_m^{-\sigma} = \lambda e^{-z}, \quad (32)$$

$$\dot{\lambda} = -\lambda \alpha_m A_m (\beta_m \omega^*)^{\alpha_m - 1}, \quad (33)$$

$$\lim_{t \rightarrow \infty} e^{-z} \lambda k = 0. \quad (34)$$

Note that in deriving (33), we use $r = \alpha_m Y_m / K_m = \alpha_m A_m (\beta_m \omega)^{\alpha_m - 1}$.

The market clearing condition of services is $c_s = Y_s$. Thus in view of the national income account,

$$rk + w = Y_m + p_a Y_a + p_s Y_s = c_m + p_a c_a + p_s c_s + \dot{k},$$

we see that the flow budget constraint of the household is written as the trade balance equation such that

$$\dot{k} = Y_m + \hat{p}_a Y_a - c_m - \hat{p}_a c_a.$$

This equation describes capital formation in the small country. Using the supply functions of manufacturing and agricultural goods as well as the demand function of services, the above

equation is rewritten as follows:

$$\begin{aligned} \dot{k} = & \frac{k - \beta_a \omega^* + (\beta_a - \beta_s) \omega^* L^s(\omega^*, c_m)}{(\beta_m - \beta_a) \omega^*} A_m (\beta_m \omega^*)^{\alpha_m} \\ & + \frac{\beta_m \omega^* - k - (\beta_m - \beta_s) \omega^* L^s(\omega^*, c_m)}{(\beta_m - \beta_a) \omega^*} \hat{p}_a A_a (\beta_a \omega^*)^{\alpha_a} \\ & - c_m - \hat{p}_a \left(\frac{\gamma_a}{p_a \gamma_m} c_m + \bar{c}_a \right), \end{aligned} \quad (35)$$

where

$$L^s(\omega^*, c_m) = \frac{1}{A_s (\beta_s \omega^*)^{\alpha_s}} \left[\frac{\gamma_s}{p_s (\omega^*) \gamma_m} c_m - \bar{c}_s \right].$$

From (27) equation (35) can be expressed as

$$\dot{k} = \mu_k(\omega^*) k - \xi(\omega^*) L^s(\omega^*, c_m) - \left(1 + \frac{\hat{p}_a \gamma_a}{p_a \gamma_m} \right) c_m + \text{constant}, \quad (36)$$

where

$$\begin{aligned} \mu_k(\omega^*) &= \frac{1}{(1 + \beta_m) \omega^*} A_m (\beta_m \omega^*)^{\alpha_m} > 0, \\ \xi(\omega^*) &= \frac{1 + \beta_a}{1 + \beta_m} A_m (\beta_m \omega^*)^{\alpha_m} > 0. \end{aligned}$$

Since $L^s(\omega^*, c_m)$ increase with c_m , the right hand side of (36) increases with k and decreases with c_m . On the other hand, from (32) and $\dot{z} = \rho(c_m)$, the behavior of the c_m is described by

$$\dot{c}_m = \frac{c_m}{\sigma} \left[a_m A_m (\beta_m \omega^*)^{\alpha_m - 1} - \rho(c_m) \right]. \quad (37)$$

To sum up, the dynamic behavior of the small country in the region of non-specialization consists of (36) and (37).

Given our assumption of $\rho'(c_m) > 0$, we find that if the small-open economy has an interior steady state, it satisfies the saddle point stability. As Figure 1 shows, the stable saddle path has a positive slope. (In this figure the feasible phase space is restricted by conditions (29) and (30).) Therefore, if the initial capital stock of the small country is lower than the rest of the world, during the transition to the steady state, both k and c_m of the small country continue rising. Notice that, as (19), (20) and (21) demonstrate, when c_m and k grow, both L_m and

L_s increase, while L_a falls. Since the relative prices are determined in the world market, the value added share as well as consumption expenditure of each good exhibit the same pattern of change as the labor share of each sector shows. Consequently, the structural transformation of our small country mimics the empirical finding of the developing country as long as the economy start with a low level of capital stock.⁹

4.2 Specialization

(i) *Specialization in the agricultural goods:*

In this case $L_m = 0$, so that the small country produces the agricultural goods and services. From the full-employment condition, $K_a + K_s = K$ and $L_a + L_s = 1$, the domestic labor allocations to the second and third sector are:

$$L_a = \frac{k - k_s}{k_a - k_s}, \quad L_s = \frac{k_a - k}{k_a - k_s}.$$

Considering the above expression and (29), we see that $L_a > 0$ and $L_m = 0$ hold if the following conditions are satisfied:

$$k + (\beta_a - \beta_s) \omega^* L^s(\omega^*, c_m) > \beta_a \omega^*,$$

$$k > k_s = \beta_s \omega.$$

where ω in the second inequality is given by (38) below. It is also to be noted that in this situation, the given world price \hat{p}_a fails to fix the factor price ratio, ω , in the small country.

The supply functions of the agricultural goods and services are respectively written as

$$Y_a = \frac{k - \beta_s \omega}{(\beta_a - \beta_s) \omega} A_a (\beta_a \omega)^{\alpha_a} = y^a(k, \omega),$$

$$Y_s = \frac{\beta_a \omega - k}{(\beta_a - \beta_s) \omega} A_s (\beta_s \omega)^{\alpha_s} = y^s(k, \omega).$$

⁹As assumed by Atkeson and Kehoe (2000), if the time discount rate is constant, c_m follows $\dot{c}_m = (c_m/\omega)(r^* - \rho)$. Since r^* is fixed in the world market and it satisfies $r^* = \rho$, the optimal level of c_m in the small country stays constant over time. This means that the initial level of c_m is determined to establish $\dot{k} = r^*l + w^* - c_m - \hat{p}_a c_a = 0$. As a result, in the non-specialization regime, the small country should stay at her initial position and never catches up with the rest of the world. We have assumed endogenous time preference to avoid such a knife-edge conclusion. Kawagishi and Mino (2013) re-examine the Atkeson-Kehoe model by introducing the Uzawa-Koopmans type of endogenous time preference.

Given our assumption, it holds that

$$\begin{aligned} y_k^a(k, \omega) &> 0, & y_\omega^a(k, \omega) &< 0, \\ y_k^s(k, \omega) &< 0, & y_\omega^s(k, \omega) &> 0. \end{aligned}$$

In view of the market equilibrium condition for services, we find that capital accumulation follows

$$\dot{k} = y_a(k, \omega) - c_m - \hat{p}_a c_a.$$

The market equilibrium condition for services is

$$y^s(k, \omega) = \frac{\gamma_s}{p_s(\omega) \gamma_m} c_m - \bar{c}_s.$$

This relation gives

$$\omega = \Omega^a(k, c_m). \quad (38)$$

It can be confirmed that

$$\Omega_{c_m}^a(k, c_m) > 0, \quad \Omega_k^a(k, c_m) > 0.$$

Using (38), we see that capital accumulation in the small country is depicted by

$$\begin{aligned} \dot{k} &= y^a(k, \Omega^a(k, c_m)) - \left[1 + \frac{\gamma_s}{p_s(\Omega(k, c_m)) \gamma_m} \right] c_m - \bar{c}_s \\ &= \Lambda^a(k, c_m). \end{aligned} \quad (39)$$

We can show that $\Lambda^a(k, c_m)$ decreases with c_m and it increases with k under mild restrictions:

$$\Lambda_k^a(k, c_m) > 0, \quad \Lambda_{c_m}^a(k, c_m) < 0.$$

To derive the dynamic equation of c_m , we use

$$B(\hat{p}_a, p_s(\omega))^{1-\sigma} c_m^{-\sigma} = \lambda,$$

which leads to

$$\begin{aligned}\frac{\dot{c}_m}{c_m} &= -\frac{1}{\sigma} \left(\frac{\dot{\lambda}}{\lambda} + (1 - \sigma) \gamma_m \frac{\dot{p}_s}{p_s} \right) \\ &= \frac{1}{\sigma} \left(\frac{1}{\hat{p}_a^*} \alpha_a A_a (\beta_a \Omega^a(k, c_m))^{\alpha_a - 1} - \rho(c_m) - (1 - \sigma) \gamma_m \frac{\dot{p}_s}{p_s} \right).\end{aligned}$$

Here, using $\omega = \Omega^a(k, c_m)$, we obtain:

$$\frac{\dot{p}_s}{p_s} = \eta_c \frac{\dot{c}_m}{c_m} + \eta_k \frac{\dot{k}}{k}, \quad \eta_c > 0, \quad \eta_k > 0.$$

Consequently, the optimal level of c_m changes according to

$$\begin{aligned}\frac{\dot{c}_m}{c_m} &= \frac{1}{\sigma} \left[1 + \frac{\eta_c}{\sigma \gamma_s} \right]^{-1} \left[\frac{1}{\hat{p}_a} \alpha_a A_a (\Omega^c(k, c_m))^{\alpha_a - 1} - \rho(c_m) - \frac{1}{\gamma_s} \frac{\gamma_s \eta_k}{k} \Lambda^a(k, c_m) \right] \\ &= \Lambda^c(k, c_m)\end{aligned}\tag{40}$$

It is shown that function under mild restrictions, $\Lambda^c(k, c_m)$ satisfies

$$\Lambda_k^c(k, c_m) < 0, \quad \Lambda_{c_m}^c(k, c_m) < 0.$$

A complete dynamic system in this case consists of (39) and (40).

So far, we have assumed that the small country has the same technologies and preferences as those of the rest of the world, which means that the small country has no interior steady state inside the specialization regime. However, this is not the case if there is asymmetry in technologies and/or preferences between the home and the foreign countries. For example, suppose that the manufacturing good sector in the home country is less efficient technology of manufacturing good production, that is, A_m of the small country is less than A_m in the rest of the world. Then the small country may has a interior steady state where she specializes in agricultural goods. In this case, from the signs of partial derivatives of $\Lambda^k(k, c_m)$ and $\Lambda^c(k, c_m)$ functions, the system has a saddle point properties and the saddle path has a positive slope; see Figure 2.

(ii) *Specialization in the manufacturing goods.*

Similarly, when the home country specializes in the manufacturing goods, the production

levels of the manufacturing goods and services are respectively given by the following:

$$Y_m = \frac{k - \beta_s \omega}{(\beta_m - \beta_s) \omega} A_m (\beta_m \omega)^{\alpha_m} = y^m(k, \omega).$$

$$Y_s = \frac{\beta_m \omega - k}{(\beta_m - \beta_s) \omega} A_s (\beta_s \omega)^{\alpha_s} = y^s(k, \omega).$$

We see that

$$y_k^m(k, \omega) > 0, \quad y_\omega^m(k, \omega) < 0,$$

$$y_k^s(k, \omega) < 0, \quad y_\omega^s(k, \omega) > 0.$$

The feasible space for k and c_m in this regime should satisfy:

$$k + (\beta_m - \beta_s) \omega^* L^s(\omega^*, c_m) > \beta_m \omega^* \quad \text{and} \quad k < \beta_m \omega,$$

where ω in the second condition is given by (41) below. The relation between ω , c_m and k can be derived by the market equilibrium condition of services, $y^s(k, c_m) = c_s$, which yields

$$\omega = \Omega^m(k, c_m), \quad \Omega_k^m(k, c_m) > 0, \quad \Omega_{c_m}^m(k, c_m) > 0. \quad (41)$$

Thus capital accumulation is determined by

$$\dot{k} = y_m(k, \Omega(k, c_m)) - c_m - \hat{p}_a c_a.$$

As a consequence, the dynamic equation of capital is given by

$$\begin{aligned} \dot{k} &= \dot{k} = y^m(k, \Omega^m(k, c_m)) - \left[1 + \frac{\gamma_s}{p_s(\Omega^m(k, c_m)) \gamma_s} \right] c_m - \bar{c}_s \\ &= \Lambda^m(k, c_m), \end{aligned}$$

where

$$\Lambda_k^m(\cdot) > 0, \quad \Lambda_{c_m}^m(\cdot) < 0.$$

Additionally, the dynamic equation of c_m is

$$\frac{\dot{c}_m}{c_m} = \frac{1}{\sigma} \left[1 + \frac{\eta_c}{\sigma \gamma_s} \right]^{-1} \left[\alpha_m A_m (\Omega^m(k, c_m))^{\alpha_m - 1} - \rho(c_m) - \frac{1}{\gamma_s} \frac{\gamma_s \eta_k}{k} \Lambda^m(k, c_m) \right].$$

For example, if the small country has a higher level of A_m than the rest of the world, then the small country may have an interior steady state in the specialization regime. We can also confirm that if the small country has an interior steady state where she specializes in manufacturing goods, the steady state satisfies a saddle point property and the saddle path generally has a positive slope in (k, c_m) space.

5 Patterns of Structural Change

Let us summarize the patterns of dynamics of the small country.

$$\begin{aligned} \text{Regime I} & \left\{ \begin{array}{l} \dot{k} = y^a(\Omega^a(k, c_m), c_m) - \left[1 + \frac{\gamma_s}{p_s(\Omega(k, c_m)) \gamma_s} \right] c_m - \bar{c}_s, \\ \dot{c}_m = \frac{c_m}{\sigma} \left[1 + \frac{\eta_c}{\sigma \gamma_s} \right]^{-1} \left[\frac{1}{\hat{p}_a} \alpha_a A_a (\Omega^c(k, c_m))^{\alpha_a - 1} - \rho(c_m) - \gamma_s \eta_k \frac{\dot{k}}{k} \right], \end{array} \right. \\ \text{Regime II} & \left\{ \begin{array}{l} \dot{k} = \mu_k(\omega^*) k + \mu_c(\omega^*) c_m + \xi(\omega^*), \\ \dot{c}_m = \frac{c_m}{\sigma} \left[a_m A_m (\beta_m \omega^*)^{\alpha_m - 1} - \rho(c_m) \right]. \end{array} \right. \\ \text{Regime III} & \left\{ \begin{array}{l} \dot{k} = y^m(\Omega^m(k, c_m), c_m) - \left[1 + \frac{\gamma_s}{p_s(\Omega(k, c_m)) \gamma_s} \right] c_m - \bar{c}_s. \\ \dot{c}_m = \frac{c_m}{\sigma} \left[1 + \frac{\eta_c}{\sigma \gamma_s} \right]^{-1} \left[\alpha_m A_m (\Omega^m(k, c_m))^{\alpha_m - 1} - \rho(c_m) - \gamma_s \eta_k \frac{\dot{k}}{k} \right]. \end{array} \right. \end{aligned}$$

In the above, Regimes I and III respectively specialize to agricultural and manufacturing goods productions. Regime II is the case of non-specialization. Figure 3 depicts each regime in the k - c_m space.

First, suppose that the small country has the identical technologies and preferences as those of the rest of the world. Then the small country has a unique steady state in Regime II which is the same as the steady state of the rest of the world. Suppose further that the initial capital stock of the small country is small enough to stay in Regime I at the outset. Figure 4 displays the pattern of development of this small country in this case. This figure combines the phase

diagrams of three regimes together. In the figure, the trajectory from E_1 to E^* is the saddle path in Regime II. Then in Regime I we can find a unique path leading to point E_1 under a given level of k_0 . Therefore, on the converging equilibrium path, both k and c_m continue rising.

On the converging path, transformation of industrial structure is as follows. First, note that the value added share between the agricultural and service sectors is:

$$\frac{\hat{p}_a Y_a}{p_s Y_s} = \frac{k - \beta_s \omega}{\beta_a \omega - k} \frac{1}{p_s} \left(\frac{A_a \beta_a}{A_s \beta_m} \right) \omega^{\alpha_a - \alpha_s}$$

In the above, the relative price p_s is defined in terms of agricultural good so that $p_s = (A_a/A_s) (\beta_a^{\alpha_a}/\beta_s^{\alpha_s}) \omega^{\alpha_a - \alpha_s}$.

Thus the value added share is written as

$$\frac{\hat{p}_a Y_a}{p_s Y_s} = \frac{k - \beta_s \omega}{\beta_a \omega - k} = \frac{L_a}{L_s}.$$

Similarly, the consumption expenditure share is given by

$$\frac{\hat{p}_a c_a}{p_s c_s} = \frac{\frac{\gamma_a}{\gamma_m} c_m + \hat{p}_a \bar{c}_a}{\frac{\gamma_s}{\gamma_s} c_m - p_s \bar{c}_s}.$$

Note that from (38), when c_m and k increase, ω also rises. Hence, on the path from E_0 to E_1 , changes in the sectoral shares of value added and employment are determined by the preference as well as on the technology parameters. For example, β_s is relatively small compared to β_a , each share of the agricultural sector continue rising. In contrast, β_s is close to β_a , then the shares of service sector may increase during the transition.

In Regime II, all of the prices are exogenously fixed. Thus it is easy to confirm that on the transition path from Point E_1 to point E^* in Figure 4, the relative shares of the manufacturing and the service sector continue increasing, while the shares of the agricultural sector continues falling.

Now suppose that the technologies and/or preferences are not symmetric between the home and the foreign countries. If this is the case, the small country may have a unique steady state either in Regime I or in Regime III. If the Regime I has the steady state, then the small country ultimately has a lower capital stock than the rest of the world: the small country never catches up with the foreign countries. By contrast, if Regime III contains the steady state equilibrium, the small country accumulates a higher level of capital than the foreign countries in the long

run. Figure 5 depicts such a situation. In this figure, the path from E_2 to E^* is the saddle path in Regime III. We find that there is a unique path with a positive slope that starts from the initial point E_0 and converges to E_2 .

In Regime III, the small country produces only the manufacturing goods and services. The relative value added share between the two goods is

$$\frac{Y_m}{p_s Y_s} = \left(\frac{k - \beta_s \omega}{\beta_m \omega - k} \right) \frac{\beta_m}{\beta_s} = \frac{\beta_m (1 - L_s)}{\beta_s L_s}.$$

The relative price is given by

$$p_s = \frac{A_m \beta_m^{\alpha_m - 1}}{A_s \beta_s^{\alpha_s - 1}} \omega^{\alpha_m - \alpha_s}$$

and the consumption expenditure share is

$$\frac{c_m}{p_s c_s} = \frac{c_m}{\frac{\gamma_s}{\gamma_m} c_m - p_s \bar{c}_s}.$$

Since ω continues rising during the transition, the relative price of services increases on the path towards the steady state. The behaviors of the value added and the expenditure shares depend on the parameter conditions. If the difference in factor intensities between the two sectors, so the value of $\beta_m - \beta_s$, is sufficiently large and the relative TFPs, A_m/A_s is large enough, then the speed of increase in p_s is high. In this case both expenditure and value added shares of the manufacturing sector will decrease.

To sum up, unless the small country has a steady state in Regime I, the structural transformation of the small country displays a typical pattern: the income and employment as well as expenditure shares of agricultural sector declines in the long run, while the manufacturing and service sectors continue expanding the economy develops. Moreover, if the small country has a steady state in Regime III, the shares of manufacturing sector may decrease in this regime, while the service sector still expands. In this case, the model behavior mimics the frequently observed pattern of transformation in many developed countries: that is, the agricultural sector shrinks and the services sector expands in the process of development, while the manufacturing sector first rapidly expands but its relative shares start declining in the latter stage of development.

6 Conclusion

In this paper we have shown that a three-sector neoclassical growth model of the small-open economy may display the empirically plausible pattern of transformation of industrial structure. The key assumptions of our study are: each production sector holds a different capital intensity and the representative household has a non-homothetic utility function. The first assumption enables us to focus on capital accumulation rather than exogenous productivity change in the process of structural change. The second assumption yields the demand induced structural change. We have also demonstrated that changes in the trade pattern of the small country significantly promote structural transformation. The central message of this paper is that if we consider international trade, non-homothetic preferences and capital accumulation at the same time, the standard neoclassical growth model can provide us with a useful analytical framework for investigating growth and structural change.

When specifying the trade structure of the small country, we assume that the agricultural and manufacturing goods are tradables, while services are not internationally traded. In addition, we follow the traditional approach in which the comparative advantage of one country mainly depends on the level of capital stock and production technologies. While these assumptions are conventional, they fail to fully capture the trade structure in the real world. For example, services cover a large number of items and some of them such as transportation, information and financial services are internationally traded. Similarly, a considerable number of manufacturing goods such as structure and construction are in general nontradables. Moreover, the comparative advantage of one country are determined not only by factor endowment and production technologies but also by the quality and structure of markets in that country. Our future task is to consider a more realistic trade structure than that assumed in this paper.¹⁰

¹⁰Yano (2009) presents an insightful discussion on quality of markets. Since international divergence in market qualities can be the basis of comparative advantage in a broader sense than the conventional definition, Yano's (2009) research agenda would be relevant for investigating the relation between trade structure, growth and industrial transformation from a deeper perspective.

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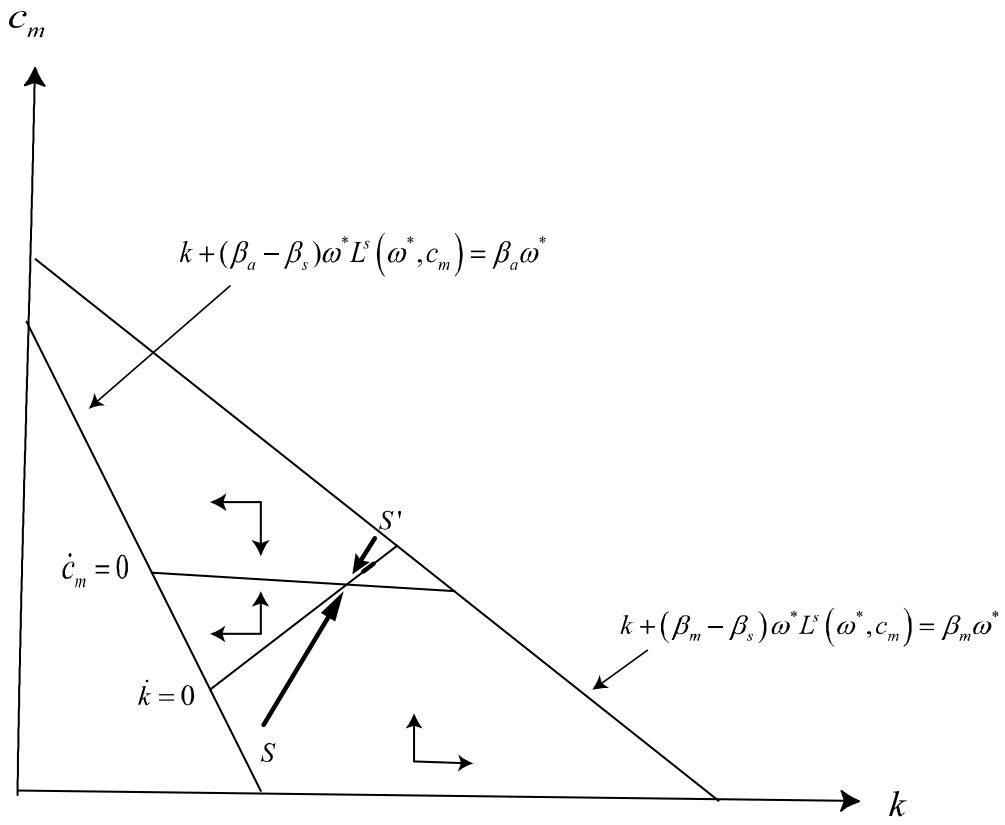


Figure 1: Equilibrium Path in the Non-Specilization Regime

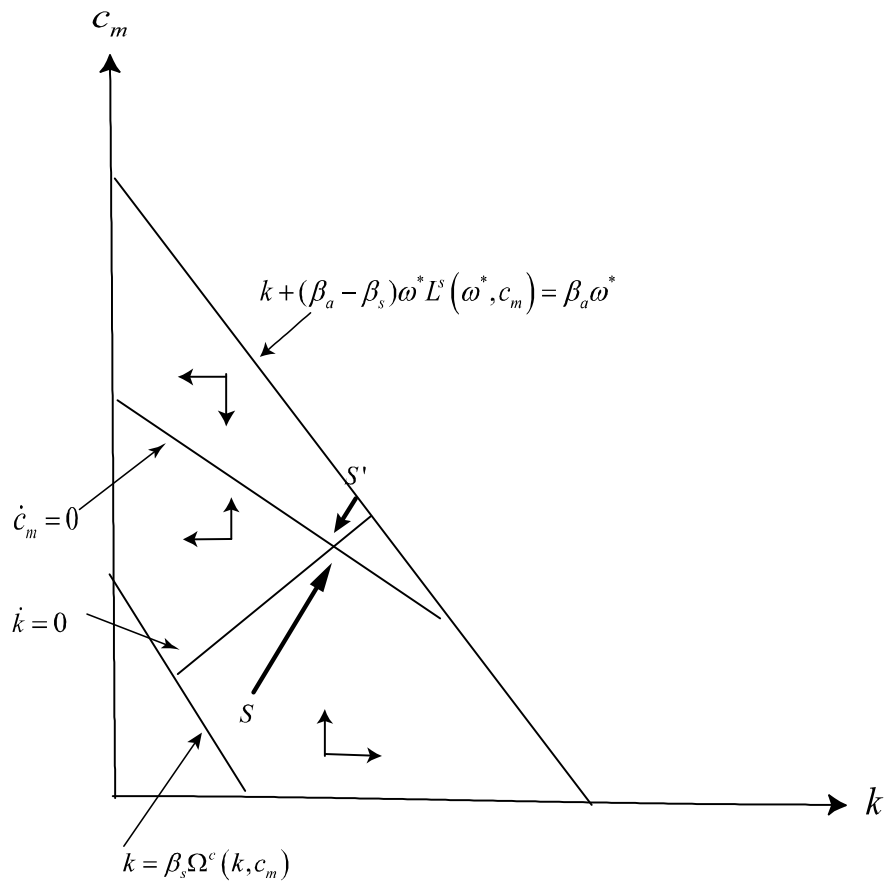


Figure 2: Equilibrium Path in the Specilization Regime (Specialize in the Agricultural Goods)

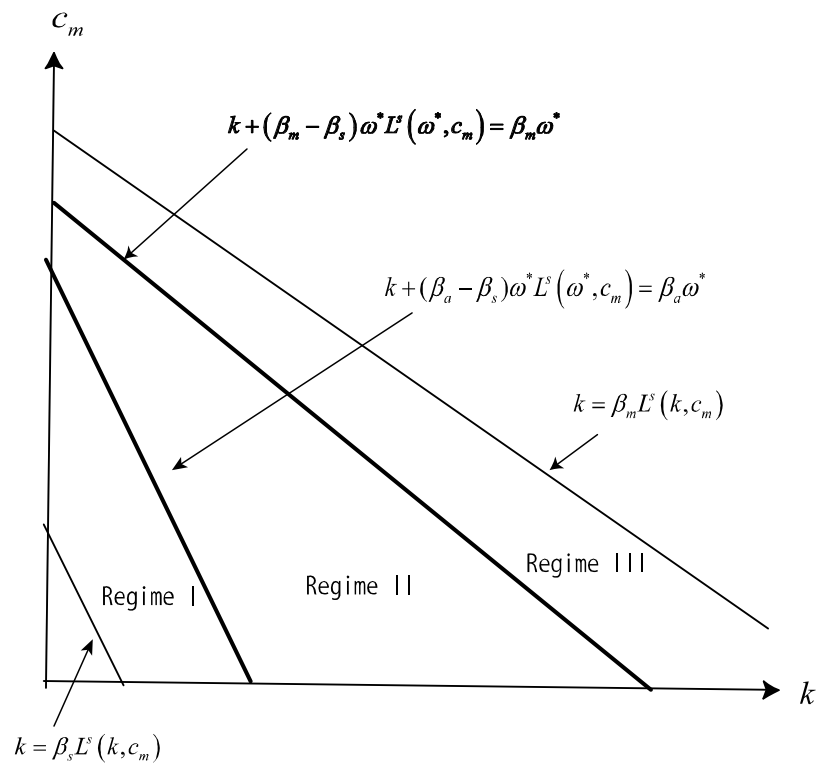


Figure 3: Classification of Regimes

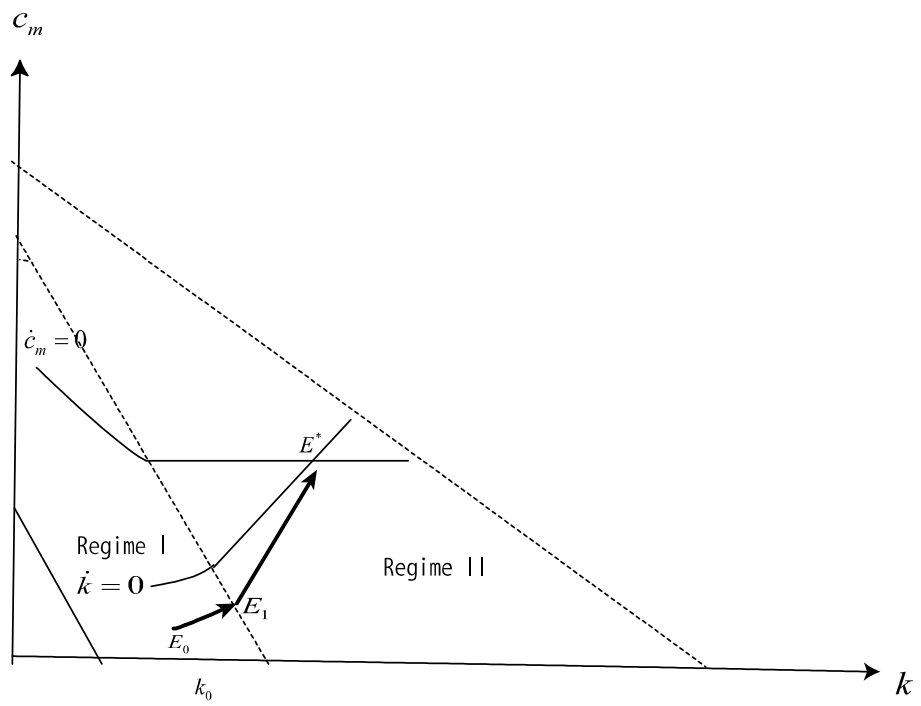


Figure 4 Convergence to the Steady State in Regime II:

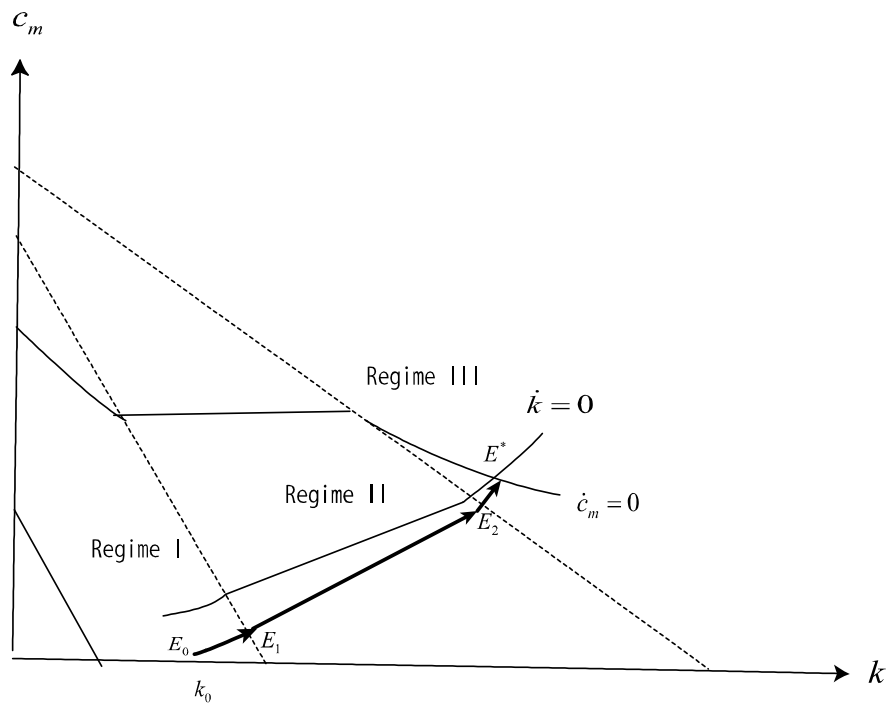


Figure 5 Convergence to the Steady State in Regime III: