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Dynamic Persuasion

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Abstract

This study develops a model of dynamic persuasion. A sender has a fixed number of arguments that contain information about the quality of his proposal, each of which is either favorable or unfavorable. The sender may try to persuade a decision maker (DM) that she has enough favorable arguments by sequentially revealing at most one argument at a time. Presenting argument is costly for the sender and delaying decisions is costly for the DM. In this dynamic game of persuasion, the sender effectively chooses when to give up persuasion and the DM decides when to make a decision. Resolving the strategic tension requires probabilistic behavior from both parties. Typically, the DM will accept the sender's proposal even when she knows that the sender's proposal may be overall unfavorable. Also, the sender's net gain from engaging in persuasion can be negative on the equilibrium path, even when persuasion is successful. We characterize the equilibrium that maximizes the DM's payoff, and perform comparative statics in the costs of persuasion on the equilibrium. We further characterize the DM's optimal stochastic commitment rule as well as the optimal non-stochastic commitment rule.

1 Introduction

Persuasion is the act of influencing someone to undertake a particular action, or, more generally, to form a certain belief. Successful persuasion takes time and is costly for both parties: the speaker exerts effort to present convincing arguments or information and, in turn, the listener reflects upon or inspects these carefully. A typical process of persuasion may involve a back-and-forth interaction where the speaker gradually presents a series of arguments up until when the listener is either sufficiently convinced by the speaker or has decided that the speaker's case lacks merit.

This paper is an attempt to understand some essential features of the dynamics of persuasion. As an example, consider an entrepreneur who is trying to convince a venture

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capitalist (VC) to invest in his startup. The VC only wants to invest if the startup is sufficiently likely to succeed. The onus is on the entrepreneur to explain and validate a number of different aspects of the project that justify investment. Of course, the VC will scrutinize each argument, possibly hiring third parties to do so. In a stylized way, the process may unfold as follows: the entrepreneur presents a set of facts about the project that the VC scrutinizes, and then the VC decides to either invest, walk away, or request further explanation; and the process repeats, with the entrepreneur deciding whether to comply or give up on persuading the VC.

This study describes the dynamic process of persuasion in a formal game theoretic model. A sender (persuader, speaker) may try to persuade a decision maker (receiver, listener) that she has enough favorable arguments for his proposal by sequentially presenting arguments by paying the communication cost. He can also remain silent, which incurs no cost for him. At each period, the decision maker chooses whether to require another argument that delays her decision making, or not. Hence she chooses to require arguments as long as she can expect that there is an informational gain from doing so. We show that the equilibrium involves probabilistic decision making from both parties. The decision maker may make a decision before she gets enough information from the sender, so she may make the wrong decision.

Our model succeeds in providing some essential features of the dynamics of persuasion. Each time the sender presents a argument, the decision maker updates her belief about the sender's proposal, and may accept the proposal knowing that it may be a wrong decision. As the game proceeds, the decision maker accumulates more and more information and it enable her to make more accurate decision. A sender with enough favorable arguments continues to persuade by showing arguments until his proposal is eventually accepted. However, he may pay too much communication costs so he may lose ex-post even though he was successful at persuasion. A sender with insufficient number of favorable arguments tries to persuade the decision maker with some probability. When the decision maker judges that the cost of requiring a further argument exceeds the additional informational benefit, she accepts the proposal for sure and the game terminates.

The fact that the equilibrium involves probabilistic decision-making is explained as follows. If the decision maker does not accept the sender's proposal until a certain amount of arguments is shown, the "unfavorable" sender with insufficient number of favorable arguments never tries to persuade. However this implies that the first piece of favorable arguments screens out the unfavorable sender and the decision maker loses the incentive to check the rest of the arguments, which in turn encourages the unfavorable sender. If the decision maker cannot make a "commitment to listen", they should use mixed strategies in order to be an equilibrium.

In an equilibrium, the decision maker accepts with a strictly positive probability every time the sender presents arguments. On the other hand, silence never meets acceptance, which tells us that only a costly message has a persuasive power.[1] Obviously, the decision maker accepts the proposal for sure only after the sender shows a favorable argument.

We further characterize the set of efficient equilibria that are not Pareto dominated by

[1]Hence, even if we endow the sender with a set of cheap messages as available message, these only have the same role as silence.

other equilibria and, furthermore, the best equilibrium for the decision maker, which is unique. We show that in an efficient equilibrium, once the process of persuasion starts, the decision maker does not make the error of rejecting a proposal with sufficient number of favorable arguments. We further show that in the best equilibrium for the decision maker she requires the largest amount of favorable arguments in order to accept for certainty among all other equilibria: intuitively, increasing the amount of favorable arguments necessary for persuasion discourages unfavorable senders from trying to persuade.

The uniqueness of the best equilibrium for all parameter values enables us to pin down a reasonable benchmark on which we conduct comparative static analysis. We particularly examine the effects of two players' costs of communication on their expected payoffs and expected duration of persuasion. We show that a decrease in the costs of communication for the decision maker (delay costs) benefits her through two effects. The first one is the direct effect. The second one, which is indirect effect, benefits the decision maker by discouraging the unfavorable sender from trying to persuade. It also reduces the sender's expected payoff because it increases the expected length of time for acceptance. With respect to the effects of the cost of communication for the sender, on top of some intuitive results, a decrease in it also lengthens the expected time of acceptance.

While in the main analysis, we assume that the decision maker cannot make any form of commitment, we also characterize her optimal commitment problem. We show that the optimal commitment mechanism takes a stochastic form, in which the decision maker attaches the highest probability of acceptance to each node that prevents the unfavorable type sender from trying to persuade. Furthermore, it can be shown that this does not harm the sender relative to the (best) equilibrium, which means that the optimal stochastic commitment can be a Pareto improvement. This is because the commitment makes it possible to avoid the case in which the unfavorable sender tries, but fails, to persuade the decision maker. In this case, which necessarily happens with a positive probability in the equilibrium, both pay wasteful communication costs.

We also examine a limited commitment method in which the decision maker can make only the non-stochastic commitment of requiring a predetermined amount of arguments. We show that even this limited commitment is beneficial for the decision maker relative to the best equilibrium when the sender's communication cost is low. However, interestingly, playing the best equilibrium is better when the sender's cost is high. This result comes from the fact that the equilibrium of the game may make the sender pay more communication cost ex-post than the gain from persuasion, which allows the decision maker to extract more information from him. In contrast, in the non-stochastic commitment, the sender is perfectly knowledgeable about the outcome of the persuasion at the beginning and, hence, it is impossible to make him show a large amount of arguments.

1.1 Related Literature

Our model is most closely related to the literature on strategic communication with verifiable messages. The most important benchmark was developed by Grossman (1981) and Milgrom (1981). They study a persuasion model in which the sender is not required to tell the truth in a precise manner but cannot falsify information, and show that complete unraveling results. Verrechia (1983) incorporates a cost of information transmission for the sender to those models, and also shows that it prevents complete unravelling of information.[2] In the current study, complete unraveling does not happen because the decision maker is not willing to pay the cost of communication up to the point that full information is obtained.

One prominent difference of the current paper from the majority of the literature on strategic communication with verifiable message is that it works on a dynamic setting. In this sense, it is related to a small number of studies. Forges and Koessler (2008) characterize the sets of equilibrium payoffs achievable with unmeditated communication in persuasion games with multi-stages. Hörner and Skrzypacz (2011) study a dynamic model of verifiable information transmission in which a seller can transmit information gradually as the buyer makes payment for it. In their model, the seller controls the amount of information that he conveys at a time, while it is fixed by a communication technology in our model. Augenblick and Bodoh-Creed (2012) build a dynamic persuasion model in which agents have preference for privacy, and show that agents can do better by gradually revealing personal information.

Insert Eso and Wallace (2011), and Dierker and Subrahmanyam (2013).

There are some studies that investigate a problem of persuasion as a mechanism design problem. In Glazer and Rubinstein (2004), the decision maker is allowed to check one argument of the sender's proposal, and they study mechanisms that maximize the probability that the decision maker makes the right decision. Sher (2010) generalizes the Glazer and Rubinstein's model in a way that both static and dynamic persuasion can be considered, and characterizes the relation between them. Kamenica and Gentzkow (2010) demonstrate that a sender can induce his favorite action from the decision maker by ingeniously designing the signal structure, by which they can make Bayesian updating of information. The current study also addresses a similar problem of how the decision maker should design her acceptance rule by examining the optimal commitment problem.

Analytically, this study is closely related to a variant of the games of attrition where players use mixed strategies to resolve the dynamic strategic tension. Hendricks, Weiss, and Wilson (1988) study a war of attrition in a complete information model. Kreps and Wilson (1982b), and Ordover and Rubinsten (1986) consider models of attrition with asymmetric information. Although they build models on zero-sum payoff structure while we do not, their models are analytically similar to our model in the sense that some of the players must play mixed strategies to have a gradual revelation of types in an equilibrium. An important difference is that in their studies, one of the informed players has a dominant strategy for the duration of the game and, as a consequence, all nodes of the game are reached with a positive probability. However, in our study, no player has a dominant strategy and all players' incentives are endogenously determined in the game. One more important difference is that in the variant of war of attrition, duration works as an indirect signal about player's private information, which can be cost of fighting, cost of failing the agreement, time preference, and so on, through showing how much they can "burn money". In our model, in contrast, private information is gradually revealed by the process of the

[2] Shin (1994) also shows that unraveling of information breaks down in a persuasion game when the decision maker does not know how precise the sender's information is.

decision maker directly asking the sender. Baliga and Ely (2010) consider a model in which a principal uses torture to extract information from an informed agent. In equilibrium, the informed agent reveals information gradually, initially resisting and facing torture, but eventually he concedes.

This paper is organized as follows. Section 2 introduces the basic structure of the model. In Section 3, we provide analysis on the simplest example of the model. In Section 4, we provide general characterization of equilibrium. Section 5, we do comparative static analysis. In section 6, we examine commitment problems. Proof of the theorems can be found in the Appendix.

2 Model

There are two players: a sender (persuader) and a decision maker, or DM hereafter.[3] The sender has a proposal that he would like the DM to accept. The quality of the sender's proposal θ (the state) is either 1 or -1; there is a common prior over the state. Without observing the true state θ,[4] the sender receives $N \in \mathbb{Z}$ number of arguments that contain information about the quality of the proposal, where N is common knowledge.[5] Each argument is either favorable (f) or unfavorable (u). We assume that the arguments are interchangeable with each other. Given the assumption, we have the expected value of θ, conditional on the realization of j number of favorable arguments among N, and denote it by $\mathbb{E}[\theta|j]$. We assume that $\mathbb{E}[\theta|j]$ is increasing with j, which means that more favorable arguments makes the prospect of the proposal better for the DM. In order to exclude trivial cases, we assume that $\mathbb{E}[\theta|0] < 0$ and $\mathbb{E}[\theta|N] > 0$.

Denote by ξ the threshold number of favorable arguments that makes the expected value of θ positive, that is,

$$\mathbb{E}[\theta|\xi - 1] < 0 \leq \mathbb{E}[\theta|\xi].$$

Furthermore, denote by $f(j)$ the unconditional density over the realization of the number of favorable arguments; the sender's *type*. The DM wants to accept the sender's proposal if $\theta = 1$ and reject if $\theta = -1$ and, hence, she cares about the sender's type. Everything, except the realization of the sender's type is common knowledge.

To illustrate our setting, as an example, think of the following simple scenario. The prior probability that the state being 1 is half and there are two arguments, i.e., $N = 2$. Each argument is independent from each other. Conditional on $\theta = 1$, the probability that a realization of an argument is f is $p > 1/2$, and conditional on $\theta = -1$, it is $1 - p$. Then, it follows that

$$\mathbb{E}[\theta|0] = \frac{1-2p}{p^2 + (1-p)^2}, \ \mathbb{E}[\theta|1] = 0, \text{ and } \mathbb{E}[\theta|2] = \frac{2p-1}{p^2 + (1-p)^2},$$

[3] Throughout, we use female pronouns for the decision maker and male pronouns for the sender.

[4] In our model, it does not matter at all whether we assume that the sender observes θ or not.

[5] We can relax this assumption without changing anything, as long as $f(j)$ and $\mathbb{E}[\theta|j]$ for $i \leq \xi$ are kept being common knowledge.

Hence, $\mathbb{E}[\theta|2] > \mathbb{E}[\theta|1] > \mathbb{E}[\theta|0]$ and ξ is one. Also, $f(0) = f(2) = \frac{1}{2}\{p^2 + (1-p)^2\}$ and $f(1) = 1 - p^2 - (1-p)^2$. Our setting allows more general cases in a sense that we do not necessarily assume that each argument is independent from others.

2.1 Dynamic Game of Persuasion

In the process of the game, the decision maker's turn and the sender's turn alternate. At each turn of the DM, she has three choices; whether she accepts, rejects, or continues, which is interpreted as requiring to present an argument. At each turn of the sender, he has two choices; he presents a favorable argument, presents an unfavorable argument, or remains silent. It is assumed that the sender cannot reveal more than one argument at a time, which is understood to be a technological constraint of communication.[6] The game is terminated once the DM chooses to accept or reject.

The formal description of the model is as follows. Time is discrete and extends from 0 to ∞ that is denoted by $t \in T = \{0, 1, 2, .., \infty\}$. Before everything starts, Nature draws the sender type j according to f, and conditional on the realization, it chooses the quality of the proposal $\theta \in \{-1, 1\}$, which is not observed by neither of players. The number j is the sender's private knowledge. At period 0, the decision maker chooses one from $\{A, R, C\}$, where A, R, and C correspond to accept, reject, and continue (require to present a favorable argument), respectively. If C is chosen, the game proceeds to period 1. In period 1, first the sender chooses $m_1 \in \{f, u, S\}$, either he presents favorable argument (choosing f) or unfavorable argument (choosing u), or remaining silent (choosing S), under the condition that he can choose f only when $j \geq 1$ and u only when $j < N$.[7] Then, the communication takes place and the DM chooses one from $\{A, R, C\}$ and, in the case that C is chosen, the game proceeds to period 2. Now, in the beginning of period 2, the sender chooses $m_2 \in \{f, u, S\}$ under the condition that m_2 can be f only when $j \geq 2$ if he has shown f at period 1, and $j \geq 1$ if did not show f at period 1, and similarly for u. The rest of the game is described in the same manner. The game terminates once the DM chooses either A or R.

Communication history at period t is a sequence of elements from $\{f, u, S\}$ up to period t, and it is denoted with superscript by m^t. The set of all histories at period t is $M^t = \times_t \{f, u, S\}$, and the set of all histories is $M = \cup M^t$. Define function $N_f : M \to \{1, 2, ..\}$, $N_u : M \to \{1, 2, ..\}$, and $N_S : M \to \{1, 2, ..\}$ that represent the number of f, u, and S contained in message history m^t, respectively.[8] Obviously, we have $N_f(m^t) + N_u(m^t) + N_S(m^t) = t$. In the following analysis, the set of available message for type j sender after message history m^t is denoted by $M_j(m^t)$, that is $S \in M_j(m^t)$ for all (m^t, j) and $f \in M_j(m^t)$ if and only if $j > N_f(m^t)$ and $N - j > N_u(m^t)$. Therefore, $M_j(m^t)$ cannot contain f (u) if the sender runs out of favorable (unfavorable) argument

[6] We can also think that it is extremely costly to present multiple arguments at a time.

[7] We can also change the model by allowing sender to send a cheap message from a finite set of cheap messages, without adding any change to the results.

[8] More precisely, $N_f(m^t) = |\{k | m_k = f,\ k \leq t\}|$, $N_u(m^t) = |\{k | m_k = u,\ k \leq t\}|$, and $N_S(m^t) = |\{k | m_k = S,\ k \leq t\}|$.

to present after the history m^t.

Persuasion is costly for both players. For the DM, it is costly because it delays decision making and there are cognitive costs to understand the argument presented by the sender. For the sender, formulating or explaining argument are costly, and also he may want to make the DM make decision promptly especially when he is facing other outside opportunities. We can also take the interpretation of Dewatripont and Tirole (2005), which states that information is neither hard nor soft initially, but the degree of softness is endogenously changed. Only by combining the mutual effort of the two sides they turn the information into the hard type. [9].

The communication technology for our model is specified as follows:

Communication cost for the DM.

The cost of communication for the DM, after history m^t is represented by a function $\eta : \{f, u, S\} \times M \to \mathbb{R}$, where

$$\eta\left(f, m^t\right) > 0,\ \eta\left(u, m^t\right) > 0,\ \text{and } \eta\left(S, m^t\right) > 0 \text{ for all } m^t \in M.$$

Communication cost for the sender.

The cost of communication for the sender is represented by a function $\delta : \{f, u, S\} \times M \to \mathbb{R}$, where

$$\delta\left(f, m^t\right) > 0,\ \delta\left(u, m^t\right) > 0,\ \text{and } \delta\left(S, m^t\right) = 0 \text{ for all } m^t \in M.$$

These say that communicating an argument is costly for the DM as well as the sender. Silence is also costly for the DM, while it is not for the sender. Although it is possible to work on a model of positive silence cost for the sender, the assumption simplifies some of the mathematical expressions that appear later (see footnote ?).

Because it does not change the analysis in an essential way, we assume that communication costs are history independent and both $\eta\left(f, m^t\right) = \eta\left(u, m^t\right)$ and $\delta\left(f, m^t\right) = \delta\left(u, m^t\right)$ hold. Then we denote $\eta\left(f, m^t\right)$ and $\eta\left(u, m^t\right)$ by η, $\eta\left(S, m^t\right)$ by η_S, and $\delta\left(f, m^t\right)$ and $\delta\left(u, m^t\right)$ by δ. The total communication costs that two players have to pay depend on how many times arguments are presented, multiplied by the communication cost. To shorten the notation, we define the functions that represent the costs of communication along the communication history m^t for two players as follows:

$$C_{DM}\left(m^t\right) = \eta N_f\left(m^t\right) + \eta N_u\left(m^t\right) + \eta_S N_S\left(m^t\right) \text{ and } C_S\left(m^t\right) = \delta N_f\left(m^t\right) + \delta N_u\left(m^t\right).$$

As soon as the DM takes an action, both of the players get their respective payoffs. The DM's (expected) payoff when the sender type is j, which is denoted by $U_{DM}\left(a, j, m^t\right)$, depends on the particular action (accept or reject) taken by the DM, the type of sender, and the communication history after which the DM takes action:

$$U_{DM}\left(A, j, m^t\right) = \mathbb{E}[\theta|j] - C_{DM}\left(m^t\right) \text{ and } U_{DM}\left(R, j, m^t\right) = -C_{DM}\left(m^t\right).$$

[9] In Dewatripont and Tirole (2005), effort from both parties increases the probability of being able to make soft information hard.

When the DM accepts the proposal, her payoff depends on the sender type through the term $\mathbb{E}[\theta|j]$, which is the expected value of the proposal.[10] If the DM rejects the proposal, she has an outside option that ensures her payoff of zero, and just pays her communication cost.

The sender's payoff, which is denoted by $U_S(A, m^t)$, depends only on the particular action taken by the DM and the communication history, after which the DM takes the action:

$$U_S\left(A, m^t\right) = V - C_S\left(m^t\right) \text{ and } U_S\left(R, m^t\right) = -C_S\left(m^t\right),$$

where $V \geq \delta$. Hence if the DM accepts his proposal, the sender's payoff is V, which is the gain from persuading the DM, minus the communication cost . On the other hand, if the DM rejects his proposal, he just pays the communication cost.

It is worth noting that the cost of communication appears in the players' payoffs in an additively separable form. An alternative setting is one in which players' payoffs are discounted as time passes. This setting, however, cannot generate the type of equilibrium that we will characterize; in such a setting the sender does not have an incentive to give up persuasion because his payoff shrinks but remains to be positive.[11]

Now we define the strategies of the two players. At each period, the type j sender's (behavior) strategy is a probability measure $\alpha_j(\cdot, m^t)$ over available messages $M_j(m^t)$, where $\alpha_j(m, m^t)$ represents the probability that he chooses a particular message $m \in \{f, u, S\}$. The strategy of period 1 is denoted by $\alpha_j(\cdot, \varnothing)$. On the other hand, the DM's (behavior) strategy is a probability measures β over $\{A, R, C\}$, parameterized by m^t. Her strategy at period 0 is $\beta(\cdot, \varnothing)$.

We introduce notations and definitions to be used in the subsequent analysis. As the game proceeds, the DM's belief about the sender type evolves. Her belief, which is parametrized by communication history m^t, is represented by a function that maps a pair of history and sender type to a probability: $B_j : M \to [0, 1]$ for each j, such that $\sum_j B_j(m^t) = 1$, i.e., $B_j(m^t)$ is the probability that the DM attaches to the event that the sender type being j, after communication history m^t.

Given a sender's strategy α, we can define the probability that a particular message history is followed, that is

$$\varphi\left(m^t\right) = \sum_j f(j)\, \Pi_{\tau=1}^t \alpha_j\left(m_\tau, m^{\tau-1}\right).$$

Given the DM as well as the sender's strategy, we can define the set of communication histories that are reached with strictly positive probabilities (on-equilibrium histories)

$$\Delta = \left\{ m^t \,\middle|\, \sum_j f(j)\, \Pi_{s=1}^t \alpha_j\left(m_s, m^{s-1}\right) \cdot \beta\left(C, m^{s-1}\right) > 0 \right\}.$$

[10] Ultimately, the DM's utility depends on the state, action, and message history that is written as $U_{DM}(A, \theta, m^t) = \theta - C_{DM}(m^t)$ and $U_{DM}(R, \theta, m^t) = -C_{DM}(m^t)$.

[11] On the other hand, it is possible to model the DM's payoff in a discounted form and still get the same type of equilibrium, because even in such a setting, she faces the same trade-off between prompt decision making and information collection.

In the following analysis, we use the following notations for the ease. The notation (m^t, m) reads "a communication history such that m^t is followed by m". In particular, $(m^t, f) \in M^{t+1}$ represents communication history m^t followed by f. Furthermore, we denote by $f^t \in M^t$ the communication history at period t that contains only f.

2.2 Equilibrium

Our solution concept is that of perfect Bayesian equilibrium, as is defined in Fudenberg and Tirole (1991, Definition 8.2).[12] This requires that after each history of messages $m^t \in M$, the DM maximizes her expected payoff given her belief about sender's type and their future play of the game, and also the sender maximizes his expected payoff given the DM's strategy.

In order to formally define the equilibrium, we first define the value function of the players. In our game, the decision of each period necessarily depends upon the decisions of the next period, and that in turn depends on the decision of the following period, and so on. The value function we will define makes it possible to summarize all the information about the future play of the game that is necessary for making the current decision.

We start by defining the value function for the DM. In order to do this, let $\varphi(\cdot|m^t)$ be a probability distribution function over $\{f, u, S\}$, parameterized by $m^t \in M$, which can be interpreted as the DM's belief about next period's messages she will hear from the sender, should she continue. We say that a function $W : M \to \mathbb{R}$ is a value function for the DM given (φ, B) if, for all $m^t \in M$,

$$W\left(m^t\right) = \max\left\{\max_{a\in\{A,R\}} \sum_j B_j\left(m^t\right) U_{DM}(a, j, m^t), \sum_m \varphi(m|m^t) W\left(m_t, m\right)\right\}, \tag{1}$$

and

$$\lim_{t\to\infty} W(m^t) = -\infty \text{ for all } \{m_t\}_{t=0}^{\infty}. \tag{2}$$

The DM's value of history m^t is the higher one of the expected payoff when she makes decision immediately after message history m^t, and the expected value for waiting for one more period. The condition (2) is understood to be the counterpart of "no-Ponzi game condition" in dynamic optimization problems. In a typical formulation of a consumer's dynamic optimization problem, the no-Ponzi game condition ensures that the consumer cannot keep borrowing money over time and accumulating debt and, thereby, makes his utility arbitrary large. Condition (2) is reminiscent of that restriction, which is necessary to pin down the value function for the DM; without it, the uniqueness of the value function is not ensured.[13] Note that (2) is the same as requiring $\lim_{t\to\infty} W(m^t) = -C_{DM}\left(m^t\right)$, because

[12] Their definition is for finite multistage games. Here, instead, the game has infinite stages and, hence, the definition follows a slight generalization of it.

[13] Alternatively, we can work on a setting in which there is a sufficiently large finite terminal period T, and players end up just paying communication costs if the DM does not make decision until that point. This essentialy does not change any results, except for eliminating some possibilities of having unreasonably lengthy equilibrium path.

silence is costly for the DM and, hence, $\lim_{t\to\infty} C_{DM}(m^t) \to \infty$ for all sequence of history $\{m^t\}_{t=1}^{\infty}$.

Similarly, we can define the value function for the sender. Contrary to the value function for the DM, sender's value function should be parameterized by his type. We say that a function $V_j : M \to \mathbb{R}$ is a value function for the sender type j, given the DM's strategy β if

$$V_j\left(m^t\right) = \sum_{a\in\{A,R\}} \beta\left(a, m^t\right) U_S\left(A, m^t\right) + \beta\left(C, m^t\right) \max_{m\in M_j(m^t)} V_j\left(m^t, m\right) \tag{3}$$

and

$$\lim_{t\to\infty} V_j\left(m^t\right) = -\lim_{t\to\infty} \delta\{N_f\left(m^t\right) + N_u\left(m^t\right)\} \text{ for all } \{m^t\}_{t=0}^{\infty}. \tag{4}$$

The max operator in the right hand side of (3) subsumes the fact that the sender behaves optimally at the next period. We also have no-Ponzi game condition as well. Now we have the following lemma, which demonstrates the uniqueness of those value functions.

Lemma 1 *Given (φ, B), W is uniquely determined. Also, given β, V_j is uniquely determined.*[14]

With the above preparations, we can define the equilibrium.

Definition 1 *A tuple $(\alpha, \beta, B, \varphi)$ is a perfect Bayesian equilibrium if it satisfies the following conditions:*

E1. The optimality of the sender's strategy at every history of messages:

$$\alpha_j\left(m, m^t\right) > 0 \text{ only when } m \in \arg\max_{m\in M_j(m^t)} V_j\left(m^t, a\right).$$

E2. The optimality of the DM's strategy at every communication history:

$$\begin{aligned}
\beta\left(A, m^t\right) &> 0 \text{ only when } W\left(m_t\right) = \sum_j B_j\left(m^t\right) U_{DM}(A, j, m^t), \\
\beta\left(R, m^t\right) &> 0 \text{ only when } W\left(m_t\right) = \sum_j B_j\left(m^t\right) U_{DM}(R, j, m^t), \\
\text{and } \beta\left(C, m^t\right) &> 0 \text{ only when } W\left(m_t\right) = \sum_m \varphi(m|m^t) W\left(m_t, m\right).
\end{aligned}$$

E3. Bayes' rule for the belief of the DM: For all $m^t \in M$,

$$\varphi\left(m_{t+1}|m^t\right) = \sum_j B_j\left(m^t\right) \alpha_j\left(m_{t+1}, m^t\right),$$

and for all $m^t \in \Delta$,

$$B_j\left(m^t\right) = \frac{B_j\left(m^{t-1}\right) \alpha_j\left(m_t, m^{t-1}\right)}{\sum_n B_n\left(m^{t-1}\right) \alpha_n\left(m_t, m^{t-1}\right)} \text{ and } B_j\left(m^1\right) = \frac{f\left(j\right) \alpha_j\left(m_1, \varnothing\right)}{\sum_n f\left(n\right) \alpha_n\left(m_1, \varnothing\right)}.$$

[14] The proof of Lemma 1 is ommited but is available upon request.

The first condition E1 requires that the each time the sender chooses what action to choose, he chooses the one that maximizes his value. Note that, this must hold not only for communication histories that are reached with strictly positive probability, but also the histories that are not supposed to reach (off-equilibrium history). E2 also requires the optimality of the DM's choice at all histories. She chooses to continue only when it maximizes her value, and the same applies for the choices of accept and reject.

E3 requires that the posterior belief of the DM at each history should follow Bayes rule. It is worth noting that our definition of the PBE imposes a restriction on off-the-equilibrium beliefs, whereas in the canonical definition we impose no restriction.

The two belief functions B_j and φ play different roles in the DM's decision making. The belief function B_j, which shows the DM's assessment of the proposal at each history, is relevant for choosing whether to accept or reject, if she makes a decision immediately. On the other hand, the belief function φ, which shows the DM's beliefs about the sender's behavior at the next period, is more relevant for choosing whether to decide immediately or to continue.

We conclude this section by showing some immediate results that follow almost directly from the definition of the equilibrium. The first one says that once the sender presents sufficient number of favorable arguments, the DM accepts the proposal with certainty, and the sender remains silent afterwards (thus, such a node should be off-equilibrium). The proof is straightforward and omitted:

Claim 1 *1. In any equilibrium, for all m^t such that $N_f(m^t) \geq \xi$, $\beta(A, m^t) = 1$.*
2. In any equilibrium, for all m^t such that $N_f(m^t) \geq \xi$, $\alpha_j(S, m^t) = 1$ for all j.

Since the DM, after verifying that the sender's proposal has a sufficient number of favorable arguments, already knows that her optimal action is to accept the proposal irrespective of the realizations of the rest of arguments, she does not pay more communication costs and reveals the rest of arguments. On the other hand, knowing that the DM will accept the proposal, the sender does not present remaining arguments by incurring the communication cost.

Given an equilibrium, let Ξ be the set of on-equilibrium history of termination with acceptance, that is, $m^\tau \in \Xi$ if and only if $\beta(A, m^\tau) = 1$ and $m^\tau \in \Delta$. From the definition, $\beta(A, m^s) < 1$ for all m^s that is a sub-history of m^τ. The next result shows that the DM accepts the proposal for certain only after she is presented qualifying arguments.

Proposition 1 *For all $m^\tau = (m^{\tau-1}, m_\tau) \in \Xi$, it must hold that $m_\tau = f$.*

This follows because if there is a message history such that $(m^{\tau-1}, u) \in \Xi$ or $(m^{\tau-1}, S) \in \Xi$, even the lowest type sender among all types who may follow $m^{\tau-1}$ can get accepted at period τ, which implies that there is no screening of an unfavorable sender taking place at period τ. This contradicts the fact that the DM chooses continue after $m^{\tau-1}$.

3 An Example

This section is for the special case in which the number of arguments is two and every argument should be favorable for the expected value of the proposal becomes positive, that is, $N = \xi = 2$. Although this is a special case, it is useful for obtaining the basic intuition of our results.

The first observation is that the DM's strategy of checking two favorable arguments for sure is not supported as an equilibrium. This is because this naive strategy makes the sender type with only one favorable argument give up persuasion by silence from the beginning, because he knows that two favorable arguments are necessary to succeed. However, then it makes the DM lose incentive to continue after period 1, because she has already screened out low type sender. This implies that equilibrium necessarily involves mixing strategies.

In the following, in order to focus on the most interesting case, we impose the following assumptions.

$$\mathbb{E}[\theta|j \geq 1] \geq 0^{15} \text{ and } -\mathbb{E}[\theta|j = 1]f(1) \geq \eta f(2) + \eta_S f(1). \tag{5}$$

The second condition says that the cost of communication is low enough, compared to the loss from accepting type 1 sender's proposal. Roughly speaking, the DM is willing to pay the communication cost if she can screen out type one sender when she knows that the sender is not type 0.

The next proposition characterizes the *ex-ante* best equilibrium for the DM. In the statement of the theorem, we omit the description of off-equilibrium behaviors, because it is straightforward to specify those.

Proposition 2 *Strategies that satisfies the following is an equilibrium.*

1. Type 2 sender presents favorable arguments in a row: $\alpha_2(f, \varnothing) = \alpha_2(f, f) = 1,$ *and type 0 sender chooses silence at period 1:* $\alpha_0(S, \varnothing) = 1$.

2. Type 1 sender mixes at period 1:

$$\alpha_1(f, \varnothing) = -\frac{\eta f(2)}{(\mathbb{E}[\theta|j = 1] + \eta_S) f(1)} \text{ and } \alpha_1(S, \varnothing) = 1 - \alpha_1(f, \varnothing).$$

3. At period 2, the DM accepts if she has been presented two favorable arguments, and rejects otherwise:

$$\beta(A, f^2) = 1 \text{ and } \beta(R, m^2) = 1 \text{ if } m^2 \neq f^2.$$

4. At period 1, the DM mixes between continuing and accepting if she has been presented a favorable argument, and rejects otherwise:

$$\beta(A, f) = \delta/V,\ \beta(C, f) = 1 - \delta/V, \text{ and } \beta(R, m_1) = 1 \text{ if } m_1 \neq f.$$

5. At period 0, the DM continues if $\gamma \geq \max\{0, \mathbb{E}[\theta]\}$, *rejects if* $0 > \max\{\gamma, \mathbb{E}[\theta]\}$, *and accepts if* $\mathbb{E}[\theta] > \{\gamma, 0\}$, *where* $\gamma =$

$$\alpha_1(f, \varnothing) f(1)(\mathbb{E}[\theta|j = 1] - \eta) + f(2)(\mathbb{E}[\theta|j = 2] - \eta) - \{f(0) + (1 - \alpha_1(f, \varnothing)) f(1)\}\eta_S.$$

[15] This is rewritten as $f(1)\mathbb{E}[\theta|j = 1] + f(2)\mathbb{E}[\theta|j = 2] \geq 0$.

In second period, the DM accepts the proposal if the sender presents the second arguments again and otherwise rejects. In the first period, after checking an argument, the DM mixes between accepting and continuing. The probability that she accepts is δ/V, which makes the sender type 1 indifferent between trying persuasion by presenting a favorable argument and giving up by being silent. On the other hand, the probability that type one sender tries persuasion is set in a way that the DM is indifferent between accepting and continuing. The type one sender's trial probability $\alpha_1(f, \varnothing)$ follows from the condition[16]

$$-\frac{\alpha_1(f,\varnothing) f(1)}{\alpha_1(f,\varnothing) f(1) + f(2)} \mathbb{E}[\theta|j=1] = \frac{\alpha_1(f,\varnothing) f(1)}{\alpha_1(f,\varnothing) f(1) + f(2)} \eta_S + \frac{f(2)}{\alpha_1(f,\varnothing) f(1) + f(2)} \eta.$$

The left-hand-side, the conditional probability that the sender is type 1 after being presented an argument is multiplied by the expected loss of accepting this type, is the benefit of continuing one more time. The right hand side is the expected cost from continuing one more time, given the sender's strategy. Those two must be equal, because the DM must be indifferent between acceptance and continuing one more time.

At period zero, if the benefit from proceeding to period 1 is higher than the expected payoff from accepting or rejecting without communication, the DM chooses to continue. In such a case, γ, which is characterized in the proposition, becomes $W(\varnothing)$. Type 2 sender strictly prefers to present arguments, because he is sure to be able to persuade the DM in the end. This is so even when $2\delta > V$, so that the communication cost he ends up paying is larger than V. This implies that he is expecting "success with regret" to happen with some probability at the beginning of the game, because at period 1 after being required to show one more, his first communication cost is sunk and responding to the DM's request and showing the second becomes optimal.

It is worth noting that there are a lot of other equilibria in this example. For example, depending on the parameter specifications, we can also have an equilibrium that terminates at period one, after only one argument is presented. In such a case, the DM chooses to continue at time zero even if she knows that communication stops at period one and hence she is able to screen out only type zero sender. Also, we can construct an equilibrium in which the game starts with silent stages, and effective communication starts from some later period, by appropriately specifying off-equilibrium beliefs. However, the equilibrium characterized in Proposition 1 attains highest possible expected payoff for the DM (this fact is generalized in the later section).

While we can do several comparative statics on the equilibrium, here we see the effect of the change in DM's communication cost on her expected payoff (and relegate the other

[16] Alternatively, we can write it as

$$\begin{aligned}\frac{f(2)}{\alpha_1(f,\varnothing) f(1) + f(2)} \mathbb{E}[\theta|j &= 2] + \frac{\alpha_1(f,\varnothing) f(1)}{\alpha_1(f,\varnothing) f(1) + f(2)} \mathbb{E}[\theta|j=1] \\ &= \frac{f(2)}{\alpha_1(f,\varnothing) f(1) + f(2)} (\mathbb{E}[\theta|j=2] - \eta) + \frac{\alpha_1(f,\varnothing) f(1)}{\alpha_1(f,\varnothing) f(1) + f(2)} \eta_S,\end{aligned}$$

where the left hand side represents the DM's expected payoff from accepting the sender's proposal after checking a favorable argument, while the right hand side is her expected payoff from continue and screen type one sender out.

comparative statics on the later section). It is obvious that η has a strictly negative relationship with $W(\varnothing)$. An interesting fact is that when η decreases, the DM can enjoy not only direct effect as well as indirect effect of the decrease. It is seen by the following relation:

$$\frac{\partial W(\varnothing)}{\partial \eta} = \underbrace{-\alpha_1(f,\varnothing) f(1) - f(2)}_{\text{Direct effect } (-)} + \underbrace{\frac{\partial \alpha_1(f,\varnothing)}{\partial \eta} f(1)(\mathbb{E}[\theta|j=1] - \eta + \eta_S)}_{\text{Indirect effect } (-)},$$

$$\text{where } \frac{\partial \alpha_1(f,\varnothing)}{\partial \eta} = \frac{-f(2)}{f(1)(\mathbb{E}[\theta|j=1] + \eta_S)} > 0.$$

The direct effect is obvious. Since the sender will communicate an argument with probability $\alpha_1(f,\varnothing) f(1) + f(2)$ at period one, this becomes the first order effect on the decrease in η. Indirect effect stems from the fact that the DM must be indifferent between continuing and accepting at period 1. To keep her indifferent when η is smaller, the probability that type 1 sender goes to period 1 should be suppressed so that the gain from screening that type out at period 2 gets smaller. This implies that type one sender tries to persuade the DM with smaller probability, which benefits the DM.

Another important implication is that the DM wants to make commitment. To see this point, think of the following method: the DM commits to listen for two periods as long as the sender presents arguments, and she accepts the proposal if two are shown. If, on the other hand, the sender chooses silence, the DM immediately rejects the proposal. To ensure that the sender's incentive compatibility is satisfied, we assume that $V \geq 2\delta$.[17] Then the expected utility for the DM from this limited method of commitment is $f(2)(\mathbb{E}[\theta|2] - 2\eta) - (f(0) + f(1))\eta_S$. With probability $f(2)$, the sender is type 2 and the DM has to pay the communication cost of 2η. Otherwise, the sender is a bad type (type 0 or 1) and she will pay just a period cost of silence.

It is computed that the difference between the expected payoff from this type of commitment and the expected payoff from playing the original game is $\alpha_1(f,\varnothing) f(1)\eta$. This is because if the sender is type 1, she presents an argument with probability $\alpha_1(f,\varnothing)$ and will end up being silent in the next period. This incurs the DM the cost of η in the initial period, which can be avoided by the commitment: succinctly, the commitment makes it possible to avoid talking with type one sender, by discouraging it from the beginning. Observe that if the DM can commit to accept at period one with probability slightly smaller than δ/V but higher than 0, she can induce the same behavior from the sender and makes herself even better.

4 General Analysis

This section is for characterizing the properties of equilibria. In the first subsection, we give some basic properties of all equilibria. In the second subsection, we examine the properties that must be satisfied in an efficient equilibrium. In the third subsection, we

[17] The problem when the condition $V \geq 2\delta$ is violated is discussed in Section 6.

characterize the best equilibrium for the DM, which is essentially unique. We show that the best equilibrium requires the largest number of favorable arguments to be shown in order to sufficiently convince the DM.

4.1 Properties of All Equilibria

As in most signaling games, our model also has a plethora of equilibria. However, it is possible to identify some important properties that all equilibria have to share. The following theorem characterizes the most important properties of the equilibrium, which demonstrates that every time a favorable or unfavorable arguments is presentd on-equilibrium, the DM must accept immediately with a strictly positive probability.

Theorem 1 *In any equilibrium, for any* $m^{t+1} = (m^t, f) \in \Delta$, *it holds that* $\beta(A, m^{t+1}) \in [\delta/V, 1]$ *and* $\beta(R, m^{t+1}) = 0$. *Also, for any* $m^{t+1} = (m^t, u) \in \Delta$, *it holds that* $\beta(A, m^{t+1}) = \delta/V$ *and* $\beta(R, m^{t+1}) = 0$.

This result follows from the fact that presenting an argument incurs cost for the sender. If the probability of acceptance is very small right after (m^t, f), for the sender, presenting an arguments does not pay from myopic point of view, which implies that he expects acceptance with high probability in the future. It implies that every on-equilibrium history afterwards reaches a node that the DM perceives acceptance to be optimal. However, then she should accepts immediately rather than continuing and paying communication cost.[18]

An important implication of the theorem is that in an equilibrium, the DM must not strictly prefer to continue each time she is presented a favorable or unfavorable arguments, which implies that the benefit of screening out unfavorable sender by continuing must at least as large as the cost of doing it. This implies that the value of a message history for the DM after a argument is shown (not silence) is determined to be the expected payoff from accepting the proposal. Note, however, that it is possible that the DM strictly prefers to continue at period 0, at the point where the sender has not yet paid the communication cost.

The next statement is an immediate corollary to Theorem 1, but also provides an important characterization of the equilibrium in our game. It says that silence has essentially no power of persuading the DM.

Theorem 2 *In an equilibrium, for any* $m^{t+1} = (m^t, S) \in \Delta$, *it holds that* $\beta(A, m^{t+1}) = 0$.

From Theorem 1, every essential communication (not silent) may meet immediate acceptance. Moreover, if silence may meet immediate acceptance, acceptance is optimal for

[18] In the alternative setting in which $\eta_S = 0$, the proposition can be rewritten as follows: if $m^{t+1} = (m^t, G) \in \Delta$ then there is a sequence of silence stage $m^\tau_{t+1} = (S, .., S)$ such that

$$\beta\left(A, m^\tau_{t+1}\right) = \sum_{s=t}^{\tau-1} \beta\left(A, \left(m^{s+1}, m_s\right)\right) \beta\left(C, \left(m^{s+1}, m_s\right)\right)^{s-t} \geq \delta/V,$$

that is, the DM must accept the proposal with probability higher than δ/V before they communicate another evidence.

all contingencies from the previous period's point of view for the DM. However, then it is strictly better to accept earlier, which contradict the fact that the game is not terminated.

4.2 Undominated Equilibria

This subsection characterizes the set of equilibria that are not payoff-dominated by other equilibria.[19] Denote by $\mathcal{E}(\eta, \eta_S, \delta)$ the set of all equilibrium for a given pair of parameter values (η, η_S, δ). Also, we denote each value function with superscript e when we mention it in a particular equilibrium e. We define the set of undominated equilibria as follows:

Definition 2 *The set of undominated equilibria $\mathcal{P}(\eta, \eta_S, \delta) \subset \mathcal{E}(\eta, \eta_S, \delta)$ satisfies the following: If $e \in \mathcal{P}(\eta, \eta_S, \delta)$, there is no $\widehat{e} \in \mathcal{E}(\eta, \eta_S, \delta)$ such that $W^{\widehat{e}}(\varnothing) \geq W^{e}(\varnothing)$ and $\mathbb{E}[V_j^{\widehat{e}}(\varnothing)] \geq \mathbb{E}[V_j^{e}(\varnothing)]$[20], and one of the inequalities is strict.*

While we defined the set of Pareto optimal equilibria in a way that the sender's expected payoff is compared ex-ante, before the state of the world is realized, we can also define it in the interim way, in which the sender's expected payoff is compared after the state of the world is realized, i.e., the condition "$\mathbb{E}[V_j^{\widehat{e}}(\varnothing)] \geq \mathbb{E}[V_j^{e}(\varnothing)]$" is replaced by "$V_j^{\widehat{e}}(\varnothing) \geq V_j^{e}(\varnothing)$ for all j". However, all the results provided in this section are valid for whichever criteria we choose.

We have the following theorem, which demonstrates that every undomianted equilibrium has to satisfy three conditions.

Theorem 3 *In an undomianted equilibrium $e \in \mathcal{P}(\eta, \eta_S, \delta)$, the followings are satisfied.*

1. *For all $(m^t, S) \in \Delta$, it holds that $\beta(R, (m^t, S)) = 1$.*
2. *Unfavorable argument is never presented, that is, $(m^t, u) \notin \Delta$ for all $m^t \in M$.*
3. *(maybe this should be just a lemma) For all $(m^t, f) \in \Delta$, it holds that*

$$\beta\left(A, \left(m^t, f\right)\right) \in \{\delta/V, 1\} \text{ and } \beta\left(C, \left(m^t, f\right)\right) = 1 - \beta\left(A, \left(m^t, f\right)\right). \tag{6}$$

The fact that playing an equilibrium with a period of presenting an unfavorable arguments does not benefit the sender can be easily seen. Because presenting an unfavorable arguments has acceptance probability of δ/V, which is just enough to recover the communication cost, playing another equilibrium that skips such a period does not harm the sender (and it is possible to construct such an equilibrium). On the other hand, the fact that it does not benefit the DM is not as straightforward as one may think. To see this, think of an equilibrium such that an unfavorable argument should be presented. Then, the sender

[19] In this section, we ignore cases of some non-generic constellations of parameter values. More precisely, we exclude the cases in which

$$-\frac{f(j)\left(\mathbb{E}[\theta|j] + \eta_S\right)}{\sum_{k=j+1}^{N} f(k)} = \eta \text{ for some } j \leq \xi.$$

[20] Note that $\mathbb{E}[V_j^{e}(\varnothing)] = \sum_{j=0}^{N} f(j)\, V_j^{e}(\varnothing)$.

type N, who has only favorable arguments, has to drop at some period. This makes the expected cost of communication smaller at each period after that period. Accordingly, the probability that low type sender comes to that period decreases and thereby reduces the benefit of screening out low type sender. This makes it possible to increases the dropping at the initial period one, which itself benefits the DM. The question is whether this gain outweighs the loss of giving up the best type sender, $f(N)\mathbb{E}[\theta|N]$; the answer turns out to be negative (see the Appendix).

It is rather easy to see that silence should meet immediate rejection in an efficient equilibrium. From Theorem 2, the sender cannot be accepted after silence, which means that having such a period does not make him better off, while even silence is costly for the DM. Those imply that given an equilibrium that has silence that does not meet immediate rejection, it is possible to construct another equilibrium that skips such a period, which Pareto dominates the original equilibrium.

To see the reason that the probability of acceptance immediately after a favorable argument should be exactly δ/V or 1 in an efficient equilibrium, suppose that presenting a favorable arguments, for example for the third time, has acceptance probability strictly higher than δ/V, that is, $\beta(A,f) > \delta/V$. Then all the sender types who have more than three favorable arguments will present them at least three times. However, in such a case, we can make another equilibrium by making the DM accept the proposal with probability one after presenting three favorable arguments. This is an equilibrium, since the sender as well as DM's strategy in the original equilibrium remains an optimum. Now the sender is strictly better off because he can persuade the DM sooner, without harming the DM. Hence, the acceptance probability immediately after a favorable arguments is either maximized or minimized among all possible ways of constructing an equilibrium.

In an undominated equilibrium as long as the game is not terminated, all sender types higher a threshold keep presenting favorable arguments until the proposal is eventually accepted (it is the only optimal behavior given the DM's strategy). The DM never rejects the proposal of these sender types.

Figure 1 describes how the value of the DM evolves over time in a undominated equilibrium. At period 1, the sender sends either f or S and the DM's value becomes $W(f)$ and $-\eta_S$, respectively. Because, in an undominated equilibrium, only low type sender sends S at period 1, if $m_1 = S$, the DM's optimal action is to reject immediately and it results $W(S) = -\eta_S$. If $m_1 = f$, accepting the proposal is an optimal and she is indifferent between doing so and continuing. This means that $W(f)$ is the appropriately weighted average of $W(f^2)$ and $W(f,S)$, where the latter is $-\eta_S - \eta$. At the final period where the DM accepts the proposal for sure, say period τ, her value reaches $\mathbb{E}[\theta|j \geq \tau] - \tau\eta$.

It is useful to define the "length" of persuasion for an undominated equilibrium. Given a undominated equilibrium $e \in \mathcal{P}(\eta, \eta_S, \delta)$, we call the number λ such that $\beta\left(A, f^{\lambda}\right) = 1$ but $\beta(A, f^t) = \delta/V$ for all $t < \lambda$, as **the length of persuasion** and denote it by $N_f(e)$. The length of persuasion is simply the number of favorable arguments necessary to make the DM accept for sure. Note that the DM may accept the proposal sooner with some probability and hence the terminology should be understood to be an abbreviation of "maximum possible length of persuasion". Important properties of undominated equilibrium follow directly from the definition.

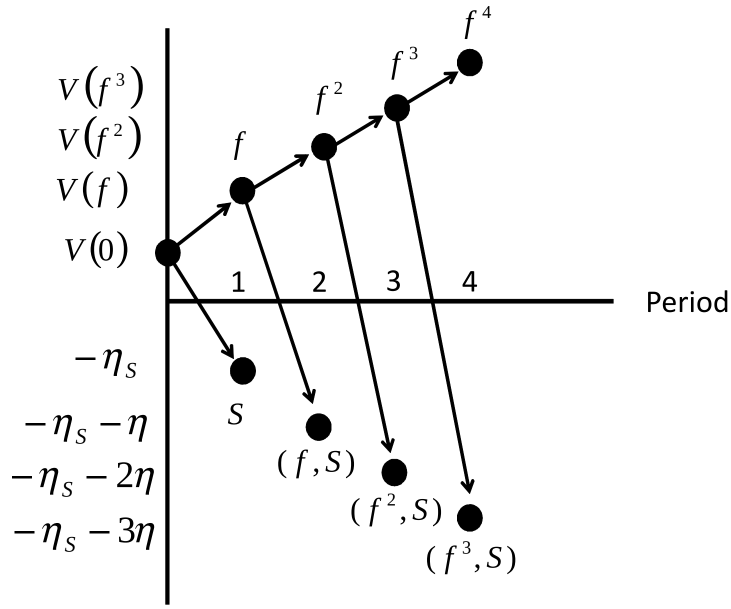

Figure 1: DM's value in an undominated equilibrium.

Claim 2 *1. In an undominated equilibrium e, for all $j < N_f(e)$, $V_j(\varnothing) = 0$. Moreover, for all $j < N_f(e)$ and $j \leq t < N_f(e)$, $V_j(f^t) = -(t-1)\delta$.*

2. In an undominated equilibrium e, for all $j \geq N_f(e)$, $V_j(\varnothing) > 0$. Moreover, for all $j \geq N_f(e)$ and $t < N_f(e)$, $V_j(f^t) > -(t-1)\delta$.

In an undominated equilibrium, after each message history, the sender has two choices; presenting a favorable arguments, or being silent.[21] Because silence effectively implies giving up persuasion, the sender's strategy is characterized by a "dropping vector". Formally, type j sender's strategy is characterized by a ξ dimensional vector

$$d_j = (d_j^1, d_j^2, ..., d_j^{\xi}),$$

where d_j^n, which is $\alpha_j(S, f^{n-1})$, represents the probability that type j sender drops persuasion by silence at n's trial, conditional on it has already shown $n-1$ favorable arguments.[22] Obviously, in an undominated equilibrium e, for all type $j \geq N_f(e)$, $d_j^n = 0$ for all $n \leq N_f(e)$, because it never drops out until eventually persuading the DM. We denote $N \times \xi$ dimensional vector $(d_1, d_2, ..., d_N)$ (collection of all sender types's strategy) by simply

[21] In a benchmark strategy equilibrium, the acceptance probability after an unfavorable argument is presented (off-equilibrium) is set to be small and, thus, silence, which incurs no cost, is better for the sender.

[22] If $j < N$, $d_j^n = 1$.

d in the subsequent analysis. The next proposition shows the equations that characterize undominated equilibrium.

Proposition 3 *Sender's strategy with dropping vector d is supported as an undominated equilibrium only if there is $\kappa \geq 1$ such that*

$$\begin{aligned} & -\sum_{j\geq t}^{\xi} d_j^{t+1}\Pi_{s=1}^{t}(1-d_j^s)f(j)\,\mathbb{E}[\theta|j] \qquad (7) \\ = \; & \eta_S\sum_{j\geq t}^{\xi} d_j^{t+1}\Pi_{s=1}^{t}(1-d_j^s)f(j)+\eta\sum_{j\geq t+1}^{N}\Pi_{s=1}^{t+1}(1-d_j^s)f(j) \end{aligned}$$

for all $t<\kappa$ and $\Pi_{s=1}^{\kappa+1}d_j^s=0$ for all $j\geq \kappa+1$, and

$$\sum_{j}^{N}\left(1-d_j^1\right)f(j)\left(\mathbb{E}[\theta|j]-\eta\right)-\eta_S\sum_{j}^{N}d_j^1 f(j)\geq 0. \qquad (8)$$

Also, any dropping vector d that satisfies above conditions are supported as an equilibrium.

In equation (7), the left-hand-side is the expected gain from screening out low type sender by continuing to period $t+1$ and thereby changes action from acceptance to rejection (note that from Theorem 1, after message history f^t, the DM's optimal action is acceptance). On the other hand, the right-hand-side is the expected cost of checking f or facing silence from unfavorable sender types. This proposition requires that these two values, when appropriately weighted, are equal with each other.

Note that (7) implies that if the DM accepts for sure at period κ, we must have

$$-\Pi_{s=1}^{\kappa-1}\left(1-d_{\kappa-1}^s\right)f(\kappa-1)\left(\mathbb{E}[\theta|\kappa-1]+\eta_S\right)=\eta\sum_{j\geq\kappa}^{N}f(j), \qquad (9)$$

because at period κ, only type $\kappa-1$ sender drops persuasion by being silent and $d_{\kappa-1}^{\kappa}=1$. Therefore, if $-\left(f(l)\,\mathbb{E}[\theta|l]+\eta_S\right)<\eta\sum_{k\geq l+1}^{N}f(k)$ for all $l\geq j$, we have no way to have an equilibrium with the maximum length of persuasion longer than $j+1$. The condition (8) ensures that after the first favorable arguments is presented, the DM's optimal action is A. If this condition is not satisfied, the DM does not accept the proposal, which contradicts Theorem 1.

We have a corollary of Theorem 1 and 3 that is used in the subsequent analysis. It determines the value of the DM's value function at the beginning of the game by a simple formula.

Corollary 1 *In an undominated equilibrium, it holds*

$$W(\varnothing)=\max\left\{0,\mathbb{E}[\theta],\sum_{j=1}^{N}(1-d_j^1)f(j)\left(\mathbb{E}[\theta|1]-\eta\right)-\sum_{j=1}^{N}d_j^1 f(j)\,\eta_S\right\} \qquad (10)$$

If $W(\varnothing) = 0$, the DM simply rejects the proposal without scrutinizing it, and if $W(\varnothing) = \mathbb{E}[\theta]$, she rubber-stamps. When these are not the case, the DM proceeds to period 1 and, hence, her value is determined by the weighted average of payoffs between the case that the sender presents a favorable arguments, where her optimal action is acceptance, and the case that he chooses silent, where her optimal action is rejection.

The way to find an undominated equilibrium is to determine the sender's strategy backward. First, we determine the final period at which the DM accepts the proposal for sure, say period κ. Second, let

$$d_j^\tau = 0 \text{ for all } j \geq \kappa \text{ and } \tau \leq \kappa,$$

that is, the sender type higher than κ certainly keeps presenting favorable arguments. It must be so in the equilibrium because for the sender type higher than κ showing a favorable arguments has a strictly higher continuation value, rather than choosing silence and being rejected. Then, we can determine $\Pi_{t=0}^{\kappa-1}\left(1 - d_{\kappa-1}^t\right)$ by (9). The rest of the values of d should be chosen in a way that (7) as well as $d_j^1 \geq 0$ for all j is satisfied. If there is no such a way of choosing d, we have no undominated equilibrium with communication. Finally, we check if

$$\sum_{j=1}^{N}(1 - d_j^1) f(j)(\mathbb{E}[\theta|1] - \eta) - \sum_{j=1}^{N} d_j^1 f(j)\,\eta_S \geq \max\{0, \mathbb{E}[\theta]\}$$

holds. If it does, we have $\beta(C, \varnothing) = 1$ and hence, we have an undominated equilibrium with the sender's dropping vector d.

4.3 The Best Equilibrium

In this subsection, we characterize the best equilibrium for the DM (the best equilibrium hereafter), that is, the equilibrium such that $W(\varnothing)$, the value of the DM at the initial period, is maximized. The result given in the previous section demonstrated that the best equilibrium, which must be a Pareto optimal equilibrium, is one of undominated equilibrium. Therefore we focus our analysis exclusively on $\mathrm{f}(\eta, \eta_S, \delta)$, the set of undominated equilibrium.

The first result demonstrates that the best equilibrium is unique for all parameter values. Thus it is possible to pin down the equilibrium on which we do comparative statics, and the highest benchmark with which the DM's equilibrium payoff is compared when we examine the commitment problem.

Proposition 4 *$\mathcal{E}(\eta, \eta_S, \delta)$ has a unique maximizer of $W(\varnothing)$ (the best equilibrium is uniquely determined).*[23]

An important characteristic of the best equilibrium for the DM is that it maximizes the length of persuasion among the set of Pareto efficient equilibria. Intuitively, increasing the amount of favorable arguments necessary for persuasion discourages unfavorable senders from trying to persuade.

[23] We regard two equilibria that are outcome equivalent identical.

Theorem 4 *If equilibrium e^* is the best equilibrium for the DM, there is no equilibrium e such that $N_f(e) > N_f(e^*)$.*

Note that there are multiple equilibria even if we focus on the ones that maximize the length of persuasion. Also, note that the theorem does not state that an equilibrium has a higher expected payoff than another equilibrium if the former has longer length of persuasion.[24] It only says that if an equilibrium is the best equilibrium, it must have the maximum length of persuasion.

To see this result in the simplest case, suppose that there are two equilibria, one with the length of persuasion of one and the other with the length of persuasion of two. In the former equilibrium, both type 1 and 2 senders try to persuade, which implies that after checking the first favorable arguments, the value of the proposal is $\mathbb{E}[\theta|j \geq 1]$ for the DM. On the other hand, in the latter equilibrium, type 1 sender does not try to persuade with probability one, which implies that after checking one argument the value of the proposal is higher than $\mathbb{E}[\theta|j \geq 1]$. Hence, the value of the decision maker at period zero is higher in the latter because it screens out more unfavorable sender (type one sender) by the first argument.

The procedure of finding the best equilibrium involves 1. find the maximum length of persuasion. 2. given the maximum length of persuasion, find the way that a low type sender is likely to remain silent at the initial period. In other words, given the length of equilibrium, we have to connect different periods by equation (7) in a way that $\left|\sum_j^{\xi-1} d_j^1 f(j)\, \mathbb{E}[\theta|j]\right|$ is maximized.

We can see the basic idea of characterizing the best equilibrium by rewriting the condition (7) as

$$-\sum_{j\geq t}^{\xi} d_j^{t+1} \Pi_{s=1}^t (1-d_j^s) f(j) \left(\mathbb{E}[\theta|j] + \eta_S\right) = \eta \sum_{j\geq t+1}^{N} \Pi_{s=1}^{t+1} (1-d_j^s) f(j)\,. \tag{11}$$

As we saw in the previous section, we can construct an equilibrium backward. We first determine the last period of persuasion, say κ, and let $d_j^t = 0$ for all $t \leq \kappa$ and $j \geq \kappa$, i.e, the sender type higher than κ never drop persuasion. Then, we choose elements of d backward so that equation (11) is satisfied for all period. At each period t, given the value of the right hand side, there are multiple ways to assign the probability of dropping, $d_j^{t+1} \Pi_{s=1}^t \left(1-d_j^s\right)$, among different types to make the equality hold. An important observation is that because the absolute value of $\mathbb{E}[\theta|j]$ decreases with j as long as $j \leq \xi$, if we decrease $d_j^{t+1} \Pi_{s=1}^t \left(1-d_j^s\right)$ a bit for some j, say by $\Delta d_j^{t+1} \Pi_{s=1}^t \left(1-d_j^s\right)$, we need to increase it for i for more than $\Delta d_j^{t+1} \Pi_{s=1}^t \left(1-d_j^s\right)$ if $i > j$. This re-allocation of the dropping probability leads to higher value of the right hand side at period $t-1$, which leads lower dropping at period 1, i.e., lower $\left|\sum_j^{\xi-1} d_j^1 f(j)\, \mathbb{E}[\theta|j]\right|$. This implies that we should use more lower types to tie the consecutive period in the equality (11).

Although it is possible to state the general algorithm to construct the best equilibrium, we provide it for the easiest case. More detailed discussion is relegated to Appendix B.

[24] This statement holds in the special case of $N = \xi = 2$, where the best equilibrium is unique.

Theorem 5 *Suppose that the function* $\Gamma : \{0, 1, .., \xi\} \to \mathbb{R}$ *defined as*

$$\Gamma(j) = \left| \frac{f(j)}{\sum_{k=j+1} f(k)} (\mathbb{E}[\theta|j] + \eta_S) \right|$$

is decreasing. Then the best equilibrium with effective communication ($\beta(C, \varnothing) > 0$)*, satisfies the followings:*

1. For $j \in \{1, .., \xi\}$, $\alpha_j(S, \varnothing) = \min\{1, c_j\}$, *where* $-\frac{c_j f(j)}{\sum_{i=j+1} c_i f(i)} (\mathbb{E}[\theta|j] + \eta_S) = \eta$, *and* $\alpha_j(S, \varnothing) = 1 - c_j$.

2. $\alpha_j(f, \varnothing) = 1$ *for all* $j \geq \xi$ *and* $\alpha_0(S, \varnothing) = 1$.

3. For t *such that* $\eta \leq \Gamma(t)$, $\beta(A, f^t) = \delta/V$, *and for* t *such that* $\eta > \Gamma(t)$, $\beta(A, f^t) = 1$.

4. If $m^t \neq f^t$, $\beta(R, m^t) = 1$.

Note that $\alpha_j(f, \varnothing)$ can be computed backward from type $\xi - 1$. From the condition that $\Gamma(j)$ is decreasing, $c_j < 1$ for some j implies $c_i < 1$ for all $i < j$. In this equilibrium, each type of sender mixes at period one whether to present an argument or to give up persuasion by being silent. Once he chooses to present an argument, he does so until he runs out of argument. At each period, say t, the DM can screen out exactly type t sender by continuing.[25]

5 Comparative Statics

In this section, we examine the effects of changes in the model's cost parameters (η, η_S, δ) on the equilibrium. In order to do this, hereafter, we focus solely on the best equilibrium for the decision maker, where value functions are denoted with superscript "*". We have the next theorem, whose proof is easy and omitted:

Theorem 6 *1. Fix* (η_S, δ). *If the prior of the proposal is low, i.e.,* $\mathbb{E}[\theta] < 0$, *then* $\mathbb{E}[V_j^*(\varnothing)]$ *is a step function of* η *and there is a threshold value of* η *under which it is increasing, and above which it is zero.*

2. Fix (η_S, δ). *If the prior of the proposal is high, i.e.,* $\mathbb{E}[\theta] > 0$, *then* $\mathbb{E}[V_j^*(\varnothing)]$ *is a step function of* η *and there is a threshold value of* η *under which it is increasing, and above which it is* V.

If the prior of the proposal is low, the DM whose communication cost η is very high does not require argument from the sender and just rejects the proposal. This is the worst case for the sender. The best case for the sender is that the DM's communication cost is low enough to require argument initially, but not too low to be willing to require much more. The sender's expected payoff is solely determined by the length of communication until the DM accepts the proposal, and because the time is discrete his expected payoff

[25] The dropping vector correspond to this equilibrium is $d_j^1 = 1 - c_j$, $d_j^k = 0$ for all $k \in \{2, .., j\}$, and $d_j^{j+1} = 1$.

becomes a step function. In this case of the prior of the proposal being unfavorable, the expected payoff of the sender is non-monotonic.

On the other hand, if the proposal is high and the DM has a very high communication cost, the DM does not require argument from the sender and just rubber-stamps the proposal. This is the best possible case for the sender. In such a case the expected payoff of the sender becomes monotonic, which is again a step function.

The effect of change in η on the DM's expected payoff is divided into direct and indirect effects. From (10), we have $\frac{\partial W^*(\varnothing)}{\partial \eta} =$

$$\underbrace{-\sum_{j=1}^{N_f(e^*)-1} \alpha_j(f,\varnothing) f(j) - \sum_{j=N_f(e^*)} f(j)}_{\text{Direct effect } (-)} + \underbrace{\sum_{j=1}^{N_f(e^*)-1} \frac{\partial \alpha_j(f,\varnothing)}{\partial \eta} f(j) (\mathbb{E}[\theta|j] - \eta + \eta_S)}_{\text{Indirect effect } (-)}.$$

We can see that the direct effect is negative, and we are able to show that the indirect effect is also negative (see Appendix). Interpretation of the direct effect is straightforward: it simply reduces the cost of communication at period one. The indirect effect comes from the later periods. The reduction in η makes it possible to make the DM indifferent between accept and continue at each period with a small benefit from screening and, hence, enable us to construct an equilibrium with a high dropping probability of unfavorable senders at the initial period.

Another important implication is that the probability that the decision maker makes a wrong decision, either choosing A when the sender type is less than ξ (type II error) or choosing R when the sender type is bigger than ξ (type I error), monotonically converges to zero as η converges to zero. Thus, by denoting the probability by $\digamma(\eta, \eta_S)$, the next theorem follows, where its proof is omitted:

Theorem 7 *Fix δ. Then,* $\lim_{\eta_{\downarrow 0}} \sup_{\eta_S \leq \eta} \digamma(\eta, \eta_S) \to 0$.

To see the theorem, note that the probability of making type II error is smaller than $\sum_{j=1}^{N_f(e^*)-1} \alpha_j(f,\varnothing) f(j)$, which is easily seen by (7) to be decreasing. On the other hand, in the best equilibrium, type I error does not occur when $\mathbb{E}[\theta] > 0$. Even when $\mathbb{E}[\theta] \leq 0$, type I error never happens as long as the DM chooses C at period 0, which is the case when η is sufficiently small.

We next consider comparative statics with respect to the sender's communication costs. It is easy to see from the construction of equilibrium that the DM's expected payoff is invariant with the sender' cost of communication. On the other hand, the sender's expected payoff can be naturally shown to be decreasing with his cost of communication.

Theorem 8 *Fix (η, η_S). Then $W^*(\varnothing)$ is constant with respect to δ and $\mathbb{E}[V_j^*(\varnothing)]$ is strictly decreasing with δ.*

The reason for $\mathbb{E}[V_j^*(\varnothing)]$ being strictly decreasing with δ is easy to see. From Claim 2, the low type sender's expected payoff is 0, irrespective of his communication cost δ, which comes from the fact that acceptance probability will adjust in an equilibrium. However, the acceptance probability at period $N_f(e^*)$, which is 1, cannot adjust with the change in δ, which implies that an increase in δ decreases the high type sender's expected payoff. These also tell that a decrease in δ lengthens the expected time before acceptance.

Remark 1 *We can also consider the equilibrium that maximizes the sender's (ex-ante) payoff. A straightforward observation is that if the prior of the proposal is high, i.e.,* $\mathbb{E}[\theta] > 0$, *it makes the DM accepts at time zero, without scrutinizing the sender's proposal. On the other hand, if the prior of the proposal is low, i.e.,* $\mathbb{E}[\theta] < 0$, *it is characterized as the equilibrium that has the shortest length of persuasion that has an on-equilibrium acceptance hisotry, if any. Those observations lead to a result similar to Dewatripont and Tirole (2005), Proposition 1. Namely, an increase in congruence between the sender and the DM* ($\mathbb{E}[\theta]$) *can lead to a breakdown of communication because, for high enough congruence, the sender can count on the DM to "rubber-stamp" his recommendation. In the neighbor of the threshold* $\mathbb{E}[\theta] = 0$, *an increase in congruence therefore raises the payoff of sender but not that of the DM.*

6 Commitment

In this section, we demonstrate that the DM can be better off by making a commitment if she can write down a contingent plan to follow. For example, in organizations, upper-level managers are frequently making decisions based on the information provided by better informed lower-level managers, and this question is particularly important for designing the rule used to handle advice. We consider two different forms of commitment.

Optimal Limited (Non-Stochastic) Commitment

We first characterize optimal commitment for the DM when she has only access to the following form of commitment: She decides to listen to the sender for a predetermined length of time, say τ, as long as arguments are kept presented. It is limited in the sense that the DM cannot make her decision in a probabilistic manner. If the DM makes a limited commitment, it is optimal for the sender types lower than τ to remain silent at period 1 and get rejected, because they know that they cannot persuade the DM. We call this type of commitment "limited commitment," hereafter.

The optimization problem the DM solves when she makes the limited commitment is:

$$\begin{aligned} \max_{\kappa} \Upsilon(k) &= \max_{\kappa} \left\{ \sum_{j \geq \kappa} f(j) \left(\mathbb{E}[\theta | j] - \kappa\eta \right) - \eta_S \sum_{j < \kappa} f(j) \right\}, \\ \text{subject to } V &\geq \kappa\delta. \end{aligned}$$

With probability $\sum_{j \geq \kappa} f(j)$, the sender is of high type and the DM has to pay the communication cost of $\kappa\eta$. Otherwise, the sender is low type and the DM pays just a period cost of silence. We have a participation constraint for the sender, $V \geq \kappa\delta$. Unless this condition is satisfied, the sender does not try to persuade the DM by paying the communication cost.

We have the following result, which shows that the DM is better off by making a limited commitment if the sender's persuasion gain V is high enough relative to his communication cost.

Theorem 9 *Suppose that* $V \geq \delta N_f(e^*)$.[26] *If* $\beta(C, \varnothing) = 1$ *in the best equilibrium, the DM prefers to make limited commitment, i.e.,* $\max_k \Upsilon(k) > W^*(\varnothing)$.

The net benefit of the limited commitment relative to the best equilibrium is that it enables her to avoid following a path in which a low type sender presents arguments initially but the DM ends up rejecting the proposal, because the sender runs out of argument. The limited commitment also makes it possible to avoid accepting a low type sender's proposal, which must occur with a strictly positive probability in the best equilibrium. However, this positive effect is cancelled out with the increased probability of continue after arguments are presented (remember that the cost of continue is equal to the expected benefit of screening out low type sender in the equilibrium).

The above result, however, can only be guaranteed if $V \geq \delta N_f(e^*)$, i.e., the sender is willing to pay the persuasion cost in order to induce his preferred action from the DM even if it takes $N_f(e^*)$ periods to present with certainty. Once this condition is violated, it is possible to have a situation where the DM prefers to play the persuasion game instead of making a limited commitment. As an example, think of the case in which $\xi = 2$ and $2\delta > V$. Then the best limited commitment is to require only one argument, because if she requires two, no sender type tries to persuade her. This gives her the same expected payoff as she plays the game with the equilibrium of $N_f(e) = 1$. However, from Theorem 4, we know that the equilibrium that attains the highest expected payoff for the DM has the longest length of persuasion, which is two.

More generally, in the best equilibrium, we may have $\delta N_f(e^*) > V$, which means that the sender presents for too long and pays more persuasion cost than what he can get (V) if the decision maker postpones the decision the most. This makes it possible for the DM to extract more information from the sender, relative to the case of limited commitment where the sender is perfectly knowledgeable about the outcome of the persuasion and, hence, never pay the communication cost excessively.

Optimal Stochastic Commitment

We can also consider the situation where the DM can make a commitment in the probabilistic way such that she accepts the proposal with a positive probability each time she is presented a favorable argument, and rejects otherwise. This type of commitment is described by a vector $(\sigma_1, ...\sigma_{\xi-1})$, where σ_j is the probability that she accepts the proposal after a favorable argument is presented for j's time. Then the problem she solves is

$$\max_{(\sigma_1,..,\sigma_{\xi-1},\alpha)} \sum_{i=0}^{\xi-1} \left[\sigma_i \Pi_{j=1}^{i-1} (1-\sigma_j)^i \sum_j \Pi_{k=1}^{i} \alpha\left(f, f^{i-1}\right) f(j)\, \mathbb{E}[\theta|j] \right]$$

$s.t$ the sender's strategy α satisfies $D1$,

[26] Note that $N_f(e^*)$ is an endogenous variable. Another sufficient condition that uses only exogenous variable is $V \geq \delta\xi$, which is stronger because $\xi \geq N_f(e^*)$.

where the sender's value function is made using the given commitment strategy; $\beta(A, f^t) = 1 - \beta(C, f^t) = \sigma_t$ for $t < \xi$, $\beta(A, f^\xi) = 1$ for $t \geq \xi$, and $\beta(R, m^t) = 1$ otherwise.

Then we can prove that the following method of commitment makes the DM better off relative to the best equilibrium: she accepts the proposal with probability δ/V each time a favorable argument is presented, until enough arguments are presented. This induces the sender not to present even a single argument, unless he has enough arguments. Knowing that, after the first period, the sender is a favorable type, the DM has an incentive to accept as soon as possible, and the probability δ/V is the largest probability of acceptance that can still make this immediate screening of sender types possible.

Theorem 10 *In the optimal commitment, the DM accepts the proposal with probability δ/V each time the sender presents an argument until κ arguments are presented, where κ is the number characterized by*

$$\begin{aligned}\kappa &= \arg\max_k \sum_{j\geq k} f(j)\,\mathbb{E}[\theta|j] \\ &\quad - \sum_{j\geq k} f(j)\left[\sum_{n\geq 1}^{k} n\eta\left(1-\frac{\delta}{V}\right)^{n-1}\frac{\delta}{V} + k\eta\left(1-\frac{\delta}{V}\right)^{\kappa-1}\right] - \eta_S \sum_{j<k} f(j)\,.\end{aligned}$$

The stochastic commitment attains additional gain from the limited commitment: it enables the DM to enjoy the benefit of avoiding accepting a low type sender's proposal, which was canceled out by the increased probability of continue and hence increased expected communication cost in the limited commitment case. The optimal stochastic commitment makes both players better off relative to the best equilibrium, because it can be shown that the length of persuasion is shorter in the commitment case than in the best equilibrium, from which combined with Claim 2, the sender can also gain.

7 Conclusion

In this study, we developed a model that describes the dynamic process of persuasion. Although we provided an entrepreneur-venture capitalist relation as a primary example, there are a lot of real world examples that fit out model. Glazer and Rubinstein (2004) provide a number of nice examples of persuasion through hard arguments.[27] Those include, for example, the case in which a worker wishes to be hired by an employer for a certain position. The worker tells the employer about his previous experience and the employer wishes to hire the worker if his ability is above a certain level.

It may be interesting to extend the model in a way that parameters η and δ, which represent players' costs of communication, have non-degenerate distributions and also are private information. Then, we will obtain more complicated strategic interactions because the fact that the game did not terminate until a particular period conveys some information

[27] They work on a setting that the DM is restricted to checking only one piece of evidence. In this sense, they think of the case where players face a very tight constraint in communication relative to our model.

about the players' types. This gives our game an additional flavor of Fudenberg and Tirole's (1986) war of attrition model.

This is a first step for a deeper understanding of the process of persuasion. There are a lot of questions that cannot be addressed in this study. These include interesting questions; which order should each argument be released when arguments have different characteristics? If the sender is allowed to show multiple arguments at a time, how does this change the nature of persuasion. Can we render a reasonable explanation for why sometimes a persuader reveals unfavorable information? Those questions are left up to future research.

8 Appendix: Proofs

In the following, we use the following notations.

$\cdot\ P(m^t)$: The set of sender types that follow history m^t with a strictly positive probabily, i.e., $j \in P(m^t)$ iff $\Pi_{s=1}^{t} \alpha_j(m_s, m^{s-1}) > 0$.

$\cdot\ \preceq$: Partial order on M defined as $m^s \preceq m^t$ if and only if $m^t = (m^s, m_{s+1}^t)$ for some m_{t+1}^s, i.e., m^t is a continuation from m^s. Asymmetric part of it is denoted by $\prec$.

$\cdot\ \rho_j(m^t)$: The probability that type j sender follows a particular communication history m^t, i.e., $\rho_j(m^t) = \Pi_{s=1}^{t} \alpha_j(m_s, m^{s-1})$.

Proof of Theorem 1: Suppose first that there is some $(m^t, f) \in \Delta$ such that $\beta(A, (m^t, f)) < \delta/V$, and fix such (m^t, f). If moreover $\beta(C, (m^t, f)) = 0$, the sender should choose S after m^t, which gives him payoff of at least $-\delta N_f(m^t)$ rather than f, which gives him payoff of only $\beta(A, (m^t, f)) \cdot V - \delta N_f(m^t) - \delta < -\delta N_f(m^t)$. Because this is a contradiction, we have $\beta(C, (m^t, f)) = 0$.

We can see that for all m'_{t+2} such that $(m^t, f, m'_{t+2}) \in \Delta$ and $\beta\left(A, (m^t, f, m'_{t+2})\right) = 0$, it holds that $\beta\left(C, (m^t, f, m'_{t+2})\right) > 0$. To see this, note that if $\beta\left(C, (m^t, f, m'_{t+2})\right) = 0$ for such m'_{t+2}, $\beta\left(R, (m^t, f, m'_{t+2})\right) = 1$ follows. Then sender type $j \in P\left(A, (m^t, f, m'_{t+2})\right)$ should have chosen S at time $t+1$ after m^t, which contradicts $(m^t, f, m'_{t+2}) \in \Delta$.

This implies that at period $t+2$, after (m^t, f), either A is chosen with a strictly positive probability or A is not chosen but C is chosen with a strictly positive probability. We can repeat the same reasoning, and see that for every $m^T \in \Xi$ such that $(m^t, f) \prec m^T$, there is m^s such that $(m^t, f) \prec m^s \preceq m^T$ and $\beta(A, m^s) > 0$. Then, it follows that

$$W\left(m^t, f\right) > \sum_{m_{t+2} \in M} \varphi\left(m_{t+2} | \left(m^t, f\right)\right) W(m^t, f, m_{t+1}),$$

which contradicts $\beta(C, (m^t, f)) > 0$. Hence $\beta(A, (m^t, f)) \geq \delta/V$ must follow. Same proof applies to show that $\beta(A, (m^t, u)) \geq \delta/V$ for all $(m^t, u) \in \Delta$.

Finally, suppose that $\beta(A, (m^t, u)) > \delta/V$ for some $(m^t, u) \in \Delta$. Then, after m^t, all the sender types can follow a continuation history that gives him strictly positive probability of acceptance, unless it has only favorable arguments. This contradicts $\beta(C, m^t) > 0$. *Q.E.D.*

Proof of Theorem 2: From Theorem 1, if there is $(m^t, S) \in \Delta$ such that $\beta(A, (m^t, S)) > 0$, A is optimal for the DM after any choices made by the sender at $t+1$ after m^t. However, this contradicts the fact that the DM chooses to continue at m^t. *Q.E.D.*

Proof of Theorem 3: Proof of Theorem 3 is done by combining several lemmata. Let $\Delta_1 \in \Delta$ be a subset of Δ such that each $m \in \Delta_1$ contains only one f, and f appears as its final element. Define Δ_j in a similar manner, that is, each $m \in \Delta_j$ contains exactly j number of f and its final element is f.

Lemma 2 *If* $\beta(A, m^t) = \beta(A, \widehat{m}^\tau)$ *for all* $m^t \in \Delta_j$ *and* $\widehat{m}^\tau \in \Delta_j$.

Proof. We first show that $\beta(A, m^t) = \beta(A, \widehat{m}^\tau)$ for all $m^t \in \Delta_1$ and $\widehat{m}^\tau \in \Delta_1$. For this purpose, suppose that there is some pair $m^t \in \Delta_1$ and $\widehat{m}^\tau \in \Delta_1$ such that $\beta(A, m^t) > \beta(A, \widehat{m}^\tau)$. Then obviously, $1 \notin P(\widehat{m}^\tau)$ follows. Also, from Theorem 1, all sender types in $P(\widehat{m}^\tau)$ presents f at least one more time after $\widehat{m}^\tau$, otherwise they strictly prefers to follow m^t than $\widehat{m}^\tau$. However, this means that at $\widehat{m}^\tau$, the DM knows that she will be presented at least one more argument. This must induce $\beta(A, \widehat{m}^{\tau+s}) = 0$ for some $\widehat{m}^{\tau+s} \in \Delta_2$ such that $\widehat{m}^{\tau+s} \succ \widehat{m}^\tau$, which contradicts Theorem 1. The lemma can be proved inductively. ■

Lemma 3 *For all* $e \in \mathcal{P}(\eta, \eta_S, \delta)$, $\beta(A, (m^t, f)) \in \{\delta/V, 1\}$ *for all* $(m^t, f) \in \Delta$.

Proof. Pick $e \in \mathcal{E}(\eta, \eta_S, \delta)$ such that the set $M^+ := \{(m^t, f) \in \Delta \mid \beta(A, (m^t, f)) \in (\delta/V, 1)\}$ is non-empty. Let M^{++} be the set of the smallest elements of M^+ with respect to the order $\prec$; that is, if $m^s \in M^{++}$ there is no $m^t \in M^+$ such that $m^t \prec m^s$. Then from Lemma 2, $N_f(m^t) = \gamma$ for some γ for all $m^t \in M^{++}$. Note that for all $m^T \in \Xi$, there is $m^t \in M^{++}$ such that $m^t \prec m^T$. Let M_1 be the set of on-equilibrium histories such that their last element is f or u and other elements are S, and also let $\lambda := \min_{m^t \in M^{++}} N_u(m^t)$. Observe that in the equilibrium, every sender type j such that $N - N_u(m^t) \geq j \geq \gamma$, follows a path in M^{++} and it holds that $V_j(\varnothing) > 0$. All other types obtain zero payoff.

Suppose that $\lambda > 0$. We will construct another equilibrium $\widehat{e} = \left(\widehat{\alpha}, \widehat{\beta}, \widehat{u}, \widehat{\varphi}\right)$ such that the set of terminal histories $\widehat{\Xi}$ is a singleton, and $\widehat{m}^T \in \widehat{\Xi}$ takes form $(u, .., u, f, .., f)$ with $N_f\left(\widehat{m}^T\right) = \gamma$ and $N_u\left(\widehat{m}^T\right) = \lambda$. Towards this end, first let $\widehat{\beta}(A, m^t) = \delta/V$ if $m^t \prec m^T$, $\widehat{\beta}(A, m^t) = 1$ for all m^t such that $N_f(m^t) \geq \xi$, and $\widehat{\beta}(R, m^t) = 1$ otherwise. Also, $\widehat{\rho}_j(m^\tau) = 1$ for all $j \in \{\gamma, ..., N - N_u(m^t)\}$. For $j \notin \{\gamma, ..., N - N_u(m^t)\}$, we choose $\widehat{\alpha}_j$ so that

$$\begin{aligned} & -\sum_{j\geq 1}^{N} \widehat{\alpha}_j\left(S, m^t\right) \widehat{\rho}_j\left(m^t\right) f(j) \left(\mathbb{E}[\theta|j] + \eta_S\right) \qquad (12) \\ = \; & \eta \sum_{j\geq 1}^{N} \sum_{m_{t+1} \in \{m_{t+1} | (m^t, m_{t+1}) \preceq m^\tau\}} \widehat{\alpha}_j\left(m_{t+1}, m^t\right) \widehat{\rho}_j\left(m^t\right) f(j), \end{aligned}$$

is satisfied for all $m^t \prec m^T$. To see that this is possible, observe that in the original equilibrium, for all history m^t such that $m^t \prec m^s$ for some $m^s \in M^{++}$, we had

$$\begin{aligned} & -\sum_{j\geq 1}^{N} \alpha_j\left(S, m^t\right) \rho_j\left(m^t\right) f(j) \left(\mathbb{E}[\theta|j] + \eta_S\right) \\ \geq \; & \eta \sum_{j\geq 1}^{N} \sum_{m_{t+1} \in \{m_{t+1} | (m^t, m_{t+1}) \preceq m^\tau\}} \alpha_j\left(m, m^t\right) \rho_j\left(m^t\right) f(j), \end{aligned}$$

because C is optimal for the DM at such m^t. Also, for all $m^t \in M^{++}$, it holds that $N_f(m^t) + N_u(m^t) \geq N_f(\widehat{m}^T) + N_u(\widehat{m}^T)$. Actually, we can choose in a way such that $\widehat{\alpha}_j(u, \varnothing) \leq \sum_{M_1} \rho_j(m^t)$ holds for all $j \notin \{\gamma, ..., N - N_u(m^t)\}$ with strict inequality for at least one j.

Now it follows that

$$\begin{aligned} 0 &\leq W^e(\varnothing) \\ &\leq \sum_{j\geq 1}^{N} \sum_{m^t \in M_1} \rho_j(m^t) f(j)(\mathbb{E}[\theta|j] - \eta) - \eta_S \sum_{j\geq 1}^{N} \sum_{m^t \in M_1} \left(1 - \rho_j(m^t)\right) f(j) \qquad (13) \\ &\leq \sum_{j\geq 1}^{N} \widehat{\alpha}_j(u, \varnothing) f(j)(\mathbb{E}[\theta|j] - \eta) - \eta_S \sum_{j\geq 1}^{N} \widehat{\alpha}_j(S, \varnothing) f(j). \end{aligned}$$

Then () and (**??**) imply that A and C are optimal for all $m^t \prec m^T$. Hence $\widehat{\beta}$ is supported as an equilibrium and right hand side of (**??**) becomes $W^{\widehat{e}}(\varnothing)$. Observe that equilibrium $\widehat{e}$ attains a strictly higher payoffs for sender and weakly for the DM. The proof for the case in which $\lambda = 0$ is similar. ■

Lemma 4 *For all $e \in \mathcal{P}(\eta, \eta_S, \delta)$, it holds that $\beta(R, (m^t, S)) = 1$ for all $(m^t, S) \in \Delta$.*

Proof. It can be proved by a similar argument as in Lemma 3 and hence ommited. ■

Lemma 5 *If there is an equilibrium e such that $(m^t, u) \in \Delta$ for some m^t, then there is another equilibrium e' that makes both the DM and the sender strictly better-off.*

Proof. *IT should be completely rewritten.* Take an equilibrium $e = (\alpha, \beta, B, \varphi)$ such that $N_u(\widehat{m}^\tau) = 1$ for some $\widehat{m}^\tau \in \Xi$. From Lemma 3, we can see that $N_u(m^\tau) = 1$ for all $m^\tau \in \Xi$, which implies that type N sender has to drop at some period. From Lemma 4, without loss of generality, we can assume that all $m^\tau \in \Xi$ does not contain S, that is, $N_f(m^\tau) + 1 = \tau$. Moreover, from Lemma 3, we can assume that for all $(m^t, f) \in \Delta$, $\beta(A, (m^t, f)) \in \{\delta/V, 1\}$. For expositional simplicity, we further assume that Ξ is a singleton. In the following, $m^t = (m_1, .., m_t)$ is mean to be a subhistory of m^τ.

Observe that $W(m_1) =$

$$\begin{aligned} &\sum_{j=1}^{N} \{1 - \alpha_j(S, \varnothing)\} f(j)(\mathbb{E}[\theta|j] - \eta) - \eta_S \sum_{j=1}^{N} \alpha_j(S, \varnothing) f(j) \\ = &\sum_{j=N_f(m^\tau)}^{N-1} f(j)\, \mathbb{E}[\theta|j] - \eta \sum_{t=1}^{\tau} \sum_{j=1}^{N} \rho_j(m^t) f(j) - \eta_S \sum_{j<N_f(m^\tau),\ j=N} f(j), \end{aligned}$$

where we used the relation

$$-\sum_{j=1}^{N} \alpha_j(S, m^t) \rho_j(m^t) f(j)(\mathbb{E}[\theta|j] - \eta_S) = \eta \sum_{j=1}^{N} \rho_j(m^{t+1}) f(j),$$

for all m_t. We will show that there exists another equilibrium such that it gives a strictly higher payoff to the DM than equilibrium e does. First think of the case in which $m_1 = u$.

Take an equilibrium $e' = (\alpha', \beta', \varphi', u')$ that has the maximal number of favorable arguments, $N_f(e')$, that is necessary to make the DM accept for sure among all the equilibria that satisfies the three conditions stated in Theorem 3.

$$\sum_{j=N_f(m^\tau)}^{N} f(j)\,\mathbb{E}[\theta|j] - \eta \sum_{t=1}^{\kappa-1}\sum_{j=1}^{N} \rho_j\left(m^t\right) f(j) - \eta_S \sum_{j<N_f(m^{\kappa-1})} f(j),$$

Now define

$$\begin{aligned} \Delta\alpha(j,t) &= \rho'_j\left(f^{t-1}\right)\left(1-\rho'_j\left(m_t, m^{t-1}\right)\right) f(j) \\ &\quad -\Pi_{s=1}^{t-1}\alpha_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right) f(j) \\ \text{and } \Delta E(j,t) &= \Pi_{s=1}^{t-1}\alpha'_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right)\mathbb{E}[\theta|j] \\ &\quad -\Pi_{s=1}^{t-1}\alpha_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right)\mathbb{E}[\theta|j] \end{aligned}$$

The proof is simpler when $N_f(e') \geq \kappa$, and hence suppose that $N_f(e') = \kappa - 1$, which implies that $-f(\kappa-1)(\mathbb{E}[\theta|\kappa-1] + \eta_S) < \eta \sum_{j=\kappa}^{N} f(j)$. Because we have

$$-\rho_{\kappa-1}\left(m^t\right) f(\kappa_{-1})(\mathbb{E}[\theta|\kappa-1] + \eta_S) = \eta \sum_{j=\kappa}^{N-1} f(j)$$

in the original equilibrium, it follows that $-\left(1-\rho_{\kappa-1}\left(m^t\right)\right) f(\kappa_{-1})(\mathbb{E}[\theta|\kappa-1] + \eta_S) < \eta f(N)$, which implies that

$$\left(\rho'_{\kappa-1}\left(f^{\kappa-1}\right) - \rho_{\kappa-1}\left(m^t\right)\right) < \frac{\eta f(N)}{\mathbb{E}[\theta|\kappa-1] + \eta_S}.$$

First, suppose that $N_f(e') \geq \tau - 2$. Define

$$\begin{aligned} \Delta\alpha(t) &= \Pi_{s=1}^{t-1}\alpha'_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right) f(j) \\ &\quad -\Pi_{s=1}^{t-1}\alpha_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right) f(j) \\ \text{and } \Delta E(j) &= \Pi_{s=1}^{t-1}\alpha'_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right)\mathbb{E}[\theta|j] \\ &\quad -\Pi_{s=1}^{t-1}\alpha_j\left(m_s, m^{s-1}\right)\left(1-\alpha'_j\left(m_t, m^{t-1}\right)\right)\mathbb{E}[\theta|j] \end{aligned}$$

If $N_f(e') = N_f(e) - 1$, because we cannot construct an equilibrium in a way that k arguments are communicated, we must have $-f(\kappa-1)(\mathbb{E}[\theta|\kappa-1] + \eta_S) < \eta \sum_{j=\kappa}^{N} f(j)$. Then because we have

$$\Pi_{s=1}^{\tau}\alpha_j\left(m_s, m^{s-1}\right) f(N_f(e)-1)(\mathbb{E}[\theta|N_f(e)-1] + \eta_S) = \eta \sum_{j\geq\kappa}^{N-1} f(j)$$

from the construction of equilibrium e, we have

$$\Delta E(\tau) < \eta f(N) \text{ and } \Delta\alpha(\tau) < \frac{1}{E(N_f(e)-1)}\eta f(N) \tag{14}$$

On the other hand, if $N_f(e') \geq \tau - 1$, (14) follows immediately. Those in turn imply that

$$\Delta E(\tau-1) < \eta f(N) + \Delta\alpha(\tau)\eta \text{ and } \Delta\alpha(\tau-1) < \frac{1}{E(N_f(e)-1)}\Delta\mathbb{E}(\tau-1).$$

By continuing this, we will get $\Delta E(t) < \eta f(N) + \sum_{s=t+1}^{\tau} \Delta\alpha(s)$ for all $j > 1$. Next we show that

$$\Delta E(t) < \eta f(N) \frac{\sum_{j\geq 1}^{N} \Pi_{s=t}^{\tau} \alpha_j(m_s, m^{s-1}) f(j)}{\sum_{j\geq\kappa}^{N-1} f(j)}. \tag{15}$$

To see this, observe that $\sum_{j\geq 1}^{N} \Pi_{s=1}^{t} \alpha_j(m_s, m^{s-1}) f(j) =$

$$\begin{aligned}
& \sum_{s=t}^{\tau}\sum_{j\geq 1}^{N} \Pi_{n=t}^{s}(1-\alpha_j(m_{s+1}, m^s))\,\alpha_j\left(m_n, m^{n-1}\right) f(j) + \sum_{j\geq\kappa}^{N-1} f(j) \\
= \; & \Gamma(t+1) + \ldots + \Gamma(\tau-1) + \Gamma(\tau),
\end{aligned}$$

where we defined $\Gamma(r) = \sum_{s=1}^{r}\sum_{j\geq 1}^{N} \Pi_{s=1}^{r}(1-\alpha_j(m_r, m^{r-1}))\,\alpha_j(m_{s-1}, m^s) f(j)$.

On the other hand we can show that

$$\begin{aligned}
\Delta E(\tau) &< \eta f(N) = \eta f(N)\frac{\sum_{j\geq\kappa}^{N-1} f(j)}{\sum_{j\geq\kappa}^{N-1} f(j)} = \eta f(N)\Gamma(\tau), \\
\Delta E(\tau-1) &< \eta f(N) + \Delta\alpha(\tau)\eta = \eta f(N)\Gamma(\tau) + \eta f(N)\frac{\eta}{\mathbb{E}(\kappa-1)} \\
&\leq \eta f(N)\Gamma(\tau) + \eta f(N)\frac{\Pi_{s=t}^{\tau-1}\alpha_{\kappa-1}(m_s, m^{s-1}) f(\kappa-1)}{\sum_{j\geq\kappa}^{N-1} f(j)} \\
&= \eta f(N)\Gamma(\tau) + \eta f(N)\Gamma(\tau-1),
\end{aligned}$$

and more generally, $\Delta E(r) < \eta f(N)\sum_{j\geq r}^{\tau}\Gamma(\tau)$, which implies

$$\Delta E(r) < \frac{\sum_{j\geq 1}^{N} \Pi_{s=1}^{r+1}\alpha_j(m_s, m^{s-1}) f(j)}{\sum_{j\geq\kappa}^{N-1} f(j)} \text{ for all } r \in \{1, .., \kappa-1\}$$

Obviously, we have $0 \leq W^e(\varnothing) - W^{e'}(\varnothing) < \sum_{j=1}^{\kappa-1}\Delta E(\tau) - \mathbb{E}(\theta|N)$. Thus (15) implies $\sum_{j\geq\kappa}^{N-1} f(j)\,\mathbb{E}(\theta|j) > \sum_{t=1}^{\tau}\sum_{j\geq 1}^{N}\Pi_{s=1}^{t}\alpha_j(m_s, m^{s-1}) f(j)$, which contradicts (**??**) since $\mathbb{E}[\theta|j]$ is increasing with j.

Next, think of the case in which we have a BSE $e' = (\alpha', \beta', \varphi', u')$ such that $N_f(e') = \kappa - 2$ but not $\kappa - 1$. The fact that we cannot construct an equilibrium in a way that $k-1$ number of arguments are presented implies

$$-f(\kappa-1)\,\mathbb{E}[\theta|\kappa-1] - f(\kappa-2)\,\mathbb{E}[\theta|\kappa-2] < 2\eta\sum_{j\geq\kappa}^{N} f(j) + \eta^2\frac{\sum_{j\geq\kappa}^{N} f(j)}{\mathbb{E}[\theta|\kappa-1]}.$$

Then by using the same argument we can get a contradiction. Other cases can be treated similarly. ■

Theorem 3 follows immediately from Lemma 3, 4, and 5. *Q.E.D.*

Proof of Proposition 3: Take an undominated equilibrium e, and let $\kappa = N_f(e)$. Because at communication history f^t such that $t < \kappa$, A as well as C are optimal for the DM, from $E1$, it must hold that

$$\sum_j B_j\left(f^t\right) U_{DM}(A, j, f^t) = \varphi(f|f^t)W\left(f^{t+1}\right) + \varphi(S|f^t)W\left(f^t, S\right).$$

In an undominated equilibrium, for all $(m, S) \in \Delta$, it holds that $W(f^t, S) = -\eta t - \eta_S$, because $\beta(R, (f^t, S)) = 1$. Moreover,

$$\begin{aligned} B_j\left(f^t\right) &= \frac{\Pi_{s=1}^t \left(1 - d_j^s\right) f(j)}{\sum_{j \geq t} \Pi_{s=1}^t \left(1 - d_j^s\right) f(j)} \\ \text{and } \varphi(f|f^t) &= \frac{\sum_{j \geq t+1} \left(1 - d_j^{t+1}\right) \Pi_{s=1}^t \left(1 - d_j^s\right) f(j)}{\sum_{j \geq t} \Pi_{s=1}^t \left(1 - d_j^s\right) f(j)}. \end{aligned} \tag{16}$$

By substituting these into (1), we can see that (7) must hold. Since the optimal action for the DM after $m^1 = f$ is A, (8) must hold as well.

In order to prove the later statement, suppose that α with dropping vector d satisfies the conditions. Let $\beta(A, f^\kappa) = 1$ and $\beta(A, f^t) = \delta/V$ for $t < \kappa$, and φ satisfies (16), as well as

$$B_{N_f(m^t)}\left(m^t\right) = 1 \text{ and } \varphi(S|m^t) = 1 \text{ for all } m^t \notin \Delta.$$

It is easily seen that E3 is satisfied.

Let the value function for the sender as follows. For sender type $j < \kappa$,

$$V_j\left(f^t\right) = -(t-1)\delta \text{ for } t < \kappa, \text{ and } V_j\left(m^t\right) = -\delta N_f\left(m^t\right) \text{ otherwise.}$$

For sender type $j \geq \kappa$,

$$V_j\left(m^t\right) = \sum_{i=0}^{\kappa-t-1} \left(\frac{\delta}{V}\right)\left(1 - \frac{\delta}{V}\right)^i (V - \delta(t+i)) + \left(1 - \frac{\delta}{V}\right)^{\kappa-t} (V - \delta\kappa),$$

for $t \leq \kappa$, $V_j(m^t) = V - \delta N_f(m^t)$ for m^t such that $N_f(m^t) \geq \kappa$, and $V_j(m^t) = \delta N_f(m^t)$ otherwise. It is straightforward to verify that V_j satisfies (3) and (4).

Take a sender type $j < \kappa$. From above, it follows that

$$V_j\left(m^t, f\right) = -\delta(t-1) = V_j\left(m^{k-1}, S\right).$$

Then he is indifferent between f and any S after $m^t = f^t$, which shows that $\alpha_j(\cdot, \cdot)$ satisfies $E2$ for $j < \kappa$. For a sender type $j \geq \kappa$, we have

$$V_j\left(f^t, f\right) > V_j\left(f^t, S\right) = -\delta t > V_j\left(f^t, u\right) = -\delta t - \delta,$$

for all $t < \kappa$, which shows that $\alpha_j(\cdot,\cdot)$ satisfies $E2$ for $j \geq \kappa$.

For the DM, let the value function be $W(m^t) = \sum_j u_j(f^t)\mathbb{E}[\theta|j] - \eta t$ for all f^t and $W(m^t) = -\eta N_f(m^t)$ for all m^t such that $m^t \neq f^t$ and $N_f(m^t) < \xi$, and $W(m^t) = \mathbb{E}[\theta|\xi] - \eta N_f(m^t)$ for all (m^t, m) such that $m^t \neq f^t$ and $N_f(m^t) \geq \xi$. It is straightforward to show that W satisfies (1) and (2).

Think about the decision at the first period. The expected payoff from not continuing is $\max\{E[\theta], 0\}$. On the other hand, the expected payoff from entering period 1 and making decision at period 1 is given by (8). Hence from E1, $\beta(C, \varnothing) = 1$ is optimal. Think about the decision after $m^t = f^t$ and $t < \kappa$. From the description of sender's strategy, we can calculate

$$W\left(f^t\right) = \mathbb{E}[U_{DM}(A, j, f^t)|f^t] - t\eta = \sum_{m\in\{f,S\}} \varphi(m|f^t) W\left(f^t, m\right),$$

follows for all $t < \kappa$. Hence $E2$ is satisfied for such f^t. It is easy to see that $E1$ is satisfied for the other cases as well, which completes the proof. *Q.E.D.*

Proof of Proposition 3 and Theorem 4: Fix model's parameter values (η, η_S, δ). It can be shown that the best equilibrium for the DM satisfies conditions stated in Proposition 3 (the proof of it is partly overlapped with the proof of Proposition 3 and hence omitted). Hence it is enough to prove that the set of undominated equilibrium has a unique maximizer of DM's value. In an undominated equilibrium, the sender's strategy α can be seen as a dropping vector $d \in [0,1]^{(\xi-1)\times\xi}$. Let $\mathcal{E}^s \subset [0,1]^{(\xi-1)\times\xi}$ be the set of the sender's strategy that is supported as an undominated equilibrium, and let $\mathcal{E}^s_\lambda \subset \mathcal{E}^s$ for $\lambda \in \{1, ..., \xi\}$ be a set of the sender's strategy that is supported by some equilibrium e such that $N_f(e) = \lambda$. We can see that $\mathcal{E}^s_\lambda$ is closed in the usual Euclidean topology, because if we take a sequence $\{\alpha^n\}_{n=1}^\infty$ from $\mathcal{E}^s$ that converges to α, (7) holds for all n and $t \leq \lambda$, which implies that the same condition holds for α from $\lim_{n\to\infty} \alpha^n \to \alpha$. Hence $\alpha \in \mathcal{E}^s_\lambda$ and $\mathcal{E}^s_\lambda$ is closed. Then $W(\varnothing)$, which is $\sum\left(1 - d^1_j\right) f(j)(\mathbb{E}[\theta|j] - \eta)$, is a continuous function on $\mathcal{E}^s_\lambda$, which is closed and bounded, and hence has a maximum point in $\mathcal{E}^s_\lambda$. Then $W(\varnothing)$ has the maximum on $\mathcal{E}^s$, which is a finite union of $\mathcal{E}^s_\lambda$.

We next prove the uniqueness. Towards this end, suppose that we have two different equilibria e and $\widehat{e}$ such that $W^e(\varnothing) = W^{\widehat{e}}(\varnothing)$.

Suppose that $N_f(e) = N_f(\widehat{e}) = \lambda$ for some λ. Then $d^t_j = \widehat{d}^t_j = 0$ for all $t \leq N_f(e)$ and $j \geq N_f(e)$. Moreover, since (9) must hold at period $\lambda - 1$, we must have $d^\lambda_{\lambda-1}\Pi^{\lambda-1}_{s=1}\left(1 - d^s_{\lambda-1}\right) = \widehat{d}^\lambda_{\lambda-1}\Pi^{\lambda-1}_{s=1}\left(1 - d^s_{\lambda-1}\right)$. Let $h < \lambda - 1$ be the largest t such that $d^t_j\Pi^{t-1}_{s=1}\left(1 - d^s_j\right) \neq \widehat{d}^t_j\Pi^{t-1}_{s=1}\left(1 - \widehat{d}^s_j\right)$ for some $j < \lambda$, and let l be the largest $j \leq h$ such that $d^h_j\Pi^{h-1}_{s=1}\left(1 - d^s_j\right) \neq \widehat{d}^h_j\Pi^{h-1}_{s=1}\left(1 - \widehat{d}^s_j\right)$. Without loss of generality, let $d^h_l\Pi^{h-1}_{s=1}\left(1 - d^s_l\right) > \widehat{d}^h_l\Pi^{h-1}_{s=1}\left(1 - \widehat{d}^s_l\right)$. Since we have (7) for period $t = h$, there must be some $q < l$ such that $d^h_q\Pi^{h-1}_{s=1}\left(1 - d^s_q\right) > \widehat{d}^h_q\Pi^{h-1}_{s=1}\left(1 - \widehat{d}^s_q\right)$. Then we can find positive numbers $\varepsilon > 0$, $\delta \geq 0$, and

$\epsilon > 0$ such that

$$\begin{aligned} & d_l^h \Pi_{s=1}^{h-1}\left(1-d_l^s\right) f(l)\left(\mathbb{E}[\theta|l]+\eta_S\right)+d_q^h \Pi_{s=1}^{h-1}\left(1-d_q^s\right) f(q)\left(\mathbb{E}[\theta|q]+\eta_S\right) \\ = & (d_l^h-\varepsilon)\{\Pi_{s=1}^{h-1}\left(1-d_l^s\right)-\epsilon\} f(l)\left(\mathbb{E}[\theta|l]+\eta_S\right) \\ & +(d_q^h+\delta)\{\Pi_{s=1}^{h-1}\left(1-d_q^s\right)-\epsilon\} f(k)\left(\mathbb{E}[\theta|q]+\eta_S\right), \end{aligned}$$

because $|\mathbb{E}[\theta|j]|$ is strictly decreasing in $j \leq \xi$.

Now think of the dropping vector $\widetilde{d}$ that satisfies the following: First, $\Pi_{s=1}^{h-1}\left(1-\widetilde{d}_j^s\right) = \Pi_{s=1}^{h-1}\left(1-d_j^s\right)$ for all $j \neq l, q$. Second,

$$\begin{aligned} \widetilde{d}_l^h &= d_l^h-\varepsilon,\ \widetilde{d}_q^h = d_q^h+\delta, \\ \Pi_{s=1}^{h-1}\left(1-\widetilde{d}_l^s\right) &= \Pi_{s=1}^{h-1}\left(1-d_l^s\right)-\epsilon, \text{ and } \Pi_{s=1}^{h-1}\left(1-\widetilde{d}_q^s\right) = \Pi_{s=1}^{h-1}\left(1-d_q^s\right)-\epsilon. \end{aligned}$$

Because $\sum_j \Pi_{s=1}^{h-1}\left(1-\widetilde{d}_q^s\right) f(j) < \sum_j \Pi_{s=1}^{h-1}\left(1-d_q^s\right) f(j)$, we can pick $\widetilde{d}_l$ and $\widetilde{d}_q^h$ in a way that (7) is satisfied for all t and $d_l^1 \leq \widetilde{d}_l^1$ and $d_q^1 \leq \widetilde{d}_q^1$. Obviously, the dropping vector $\widetilde{d}$ can be supported as an equilibrium and $W^{\widehat{e}}(\varnothing) > W^e(\varnothing)$ from Corollary 2.

Next, suppose that $N_f(e) > N_f(\widehat{e}) = \lambda$ for some λ. Because (7) holds between period λ and $N_f(e)-1$, and $\Pi_{s=1}^{\lambda}\left(1-\widehat{d}_j^s\right) = 1$ for all $j \geq \lambda$, it is possible to construct a dropping vector $\widetilde{d}$ such that (7) are satisfied between period λ and $N_f(e)-1$, and $\Pi_{s=1}^{\lambda}\left(1-\widetilde{d}_j^s\right) < \Pi_{s=1}^{\lambda}\left(1-\widehat{d}_j^s\right)$. Then we can use the similar argument as above and obtain a contradiction. *Q.E.D.*

Proof of Theorem 8: The first statement is straightforward. We have $V_j^{e^*}(\varnothing) = 0$ for $j < N_f(e^*)$, and it is

$$\sum_{s \leq N_f(e^*)-1} (V-s\delta)(1-\delta/V)^{s-1}\delta/V + (V-N_f(e)\delta)(1-\delta/V)^{N_f(e^*)-1}$$

for $j \geq N_f(e^*)$. Observe that $V_{N_f(e^*)-1}(\varnothing) =$

$$\sum_{s \leq N_f(e^*)-1} (V-s\delta)(1-\delta/V)^{s-1}\delta/V - (1-\delta/V)^{N_f(e)-1}\delta(N_f(e^*)-1),$$

because presenting f until period $N_f(e^*)-1$ is optimal for type $N_f(e^*)-1$. Since this value is zero, we have $V_j^{e^*}(\varnothing) = (1-\delta/V)^{N_f(e^*)-1}(V-\delta)$ for $j \geq N_f(e^*)$. Now obviously $\frac{\partial V_j^{e^*}(\varnothing)}{\partial \delta} < 0$ follows for $j \geq N_f(e^*)$, which implies the second statement. *Q.E.D.*

Claim: Fix (δ, η_S). Then $\sum_{j=1}^{N_f(e^*)-1} \frac{\partial \alpha_j^*(f,\varnothing)}{\partial \eta} f(j)\left(\mathbb{E}[\theta|j]-\eta+\eta_S\right) < 0$.

To obtain a contradiction, suppose that $\sum_{j=1}^{N_f(e^*)-1} \frac{\partial \alpha_j^*(f,\varnothing)}{\partial \eta}\Big|_{\eta=\eta'} f(j)\left(\mathbb{E}[\theta|1]-\overline{\eta}+\eta_S\right) > 0$ for some $\overline{\eta}$. Then, there is $\widehat{\eta} > \overline{\eta}$ sufficiently close to $\overline{\eta}$ and $\sum_{j=1}^{N_f(\widehat{e})-1}\left(1-\widehat{d}_j^1\right) f(j)\left(\mathbb{E}[\theta|1]-\right.$

$\overline{\eta}+\eta_S) \geq \sum_{j=1}^{N_f(\widehat{e})-1}\left(1-\overline{d}_j^1\right) f(j)\left(\mathbb{E}[\theta|1]-\overline{\eta}+\eta_S\right)$, where $\widehat{d}$ and $\overline{d}$ ($\widehat{d}$ and $\overline{d}$) correspond the sender' strategy in the best equilibrium (and the best equilibrium) for the DM when $\eta=\widehat{\eta}$ and $\eta=\overline{\eta}$, respectively.

We will inductively construct a dropping vector $\widetilde{d}$ that can be supported as an equilibrium when $\eta=\overline{\eta}$ and $\widetilde{d} \geq \widehat{d}$. Let the set of dropping vector

$$D^{N_f(e^*)-1} := \left\{d | d^1_{N_f(\widehat{e})-1} \geq \widehat{d}^1_{N_f(\widehat{e})-1},\ \text{and (7) is satisfied for period } N_f(\widehat{e})-1 \text{ and } \eta=\overline{\eta}\right\},$$

and

$$D^j := \left\{d | d \in D^{j+1},\ d_j^1 \geq \widehat{d}_j^1,\ \text{and (7) is satisfied for } t=j \text{ and and } \eta=\overline{\eta}\right\} \text{ for } j<N_f(\widehat{e})-1.$$

Obviously, $D^{N_f(\widehat{e})-1}$ is non-empty. Then from the fact that $\widehat{d}$ satisfies (7) for $t=N_f(\widehat{e})-2$ and $\eta=\widehat{\eta}>\overline{\eta}$ implies that $D^{N_f(\widehat{e})-2}$ is also non-empty. Continuing this argument, we can eventually obtain non-empty D^1. From the construction, D^1 is a set of equilibrium dropping vector. However, this contradicts the fact that $\overline{d}$ is the dropping vector of the best equilibrium because for all $d \in D^1$, we have

$$\begin{aligned}
\sum_{j=1}^{N_f(e^*)-1}\left(1-\widetilde{d}_j^1\right) f(j)\left(\mathbb{E}[\theta|j]-\overline{\eta}+\eta_S\right) &> \sum_{j=1}^{N_f(e^*)-1}\left(1-\overline{d}_j^1\right) f(j)\left(\mathbb{E}[\theta|j]-\overline{\eta}+\eta_S\right) \\
&\geq \sum_{j=1}^{N_f(e^*)-1}\left(1-\widehat{d}_j^1\right) f(j)\left(\mathbb{E}[\theta|j]-\overline{\eta}+\eta_S\right),
\end{aligned}$$

which implies that dropping vector $\widetilde{d}$ attains a strictly higher expected payoff than $\overline{d}$, which is a contradiction.

Q.E.D.

Proof of Theorem 9: We denote the solution to the commitment problem by r, and the length of persuasion of the best equilibrium by κ, by abbreviating their dependence on (η, η_S, δ). Since the result is trivial when $\kappa=0$, assume it is not. Let d be the corresponding dropping vector of the best equilibrium.

Because $d_j^1=0$ for all $j \geq \kappa$, we have $\Upsilon(\kappa)=\sum_{j\geq\kappa} f(j)\left(\mathbb{E}[\theta|j]-\kappa\eta\right)-\eta_S\sum_{j<\kappa} f(j)$ and $W^*(\varnothing)=\sum_{j\geq 1} f(j)(1-d_j^1)(\mathbb{E}[\theta|j]-\eta)-\eta_S\sum_{j=1} d_j^1 f(j)$. Hence $\Upsilon(\kappa)-W^*(\varnothing)$ is written as

$$-\kappa\eta\sum_{j\geq\kappa} f(j)+\sum_{j=1} f(j)\left(1-d_j^1\right)\eta-\sum_{j\geq 1}^{\kappa-1} f(n)\left(1-d_j^1\right)\mathbb{E}[\theta|j]-\eta_S\sum_{j=1}^{k-1} f(j)\left(1-d_j^1\right).$$

Then, by using (7) repeatedly, we have $\Upsilon(\kappa) - W^*(\varnothing) =$

$$\begin{aligned}
& -\kappa\eta \sum_{j\geq\kappa} f(j) + \eta \sum_{j\geq 1} f(j)\left(1-d_j^1\right) - \eta_S \sum_{j\geq 1}^{k-1} f(j)\left(1-d_j^1\right) \\
& -\sum_{j\geq 1}^{\kappa-1} f(j)\, d_j^2 \left(1-d_j^1\right) \mathbb{E}[\theta|j] - \sum_{j\geq 2}^{\kappa-1} f(j)\left(1-d_j^2\right)\left(1-d_j^1\right)\mathbb{E}[\theta|j] \\
= \; & -\eta\kappa \sum_{j\geq\kappa} f(j) + \eta \sum_{j\geq 1} f(j)\left(1-d_j^1\right) - \eta_S \sum_{j\geq 1}^{k-1} f(j)\left(1-d_j^1\right) \\
& +\eta \sum_{j\geq 1} f(j)\, d_j^2\left(1-d_j^1\right) + \eta_S \sum_{j\geq 1}^{\kappa-1} f(j)\, d_j^2 \left(1-d_j^1\right) \\
& -\sum_{j\geq 3}^{\kappa-1} f(j)\left(1-d_j^3\right)\left(1-d_j^2\right)\left(1-d_j^1\right)\mathbb{E}[\theta|j] - \sum_{j\geq 2}^{\kappa-1} f(j)\, d_j^3\left(1-d_j^2\right)\left(1-d_j^1\right)\mathbb{E}[\theta|j] \\
= \; & \cdot = -\eta\kappa \sum_{j\geq\kappa} f(j) + \eta \sum_{j\geq 1}\sum_{t\geq 1}^{\kappa} \Pi_{s=0}^{t-1}\left(1-d_j^s\right) f(j) = \eta \sum_{j\geq 1}^{\kappa-1}\sum_{t\geq 1}^{\kappa} \Pi_{s=0}^{t-1}\left(1-d_j^s\right) f(j) \geq 0,
\end{aligned}$$

where the last inequality is strict when $k \geq 1$. *Q.E.D.*

Proof of Theorem 10: We will prove that for any σ the probabilistic commitment given in the theorem attains higher expected payoff for the DM. Towards this end, pick a commitment σ and fix it. Also, denote by $\pi(\sigma)$ the DM's expected payoff associated with commitment σ, and $k(\sigma)$ be the threshold type of sender above which he is eventually accepted by the DM. It is without loss of generality to assume the followings:

$$\sum_{j\geq l+1} f(j)\,\eta < \sum_{j\geq l}^{k(\sigma)-1} f(j)\,\mathbb{E}[\theta|j] \text{ for all } l \leq k(\sigma)-1, \tag{17}$$

because otherwise, another commitment $\sigma' = (\sigma_1, .., \sigma_{l-1}, 1, .., 1)$ attains a strictly higher expected payoff for the DM.

First, suppose that $\sigma_{k(\sigma)-1} < \delta/V$. Because $\alpha_{k(\sigma)-1}\left(f, f^{k(\sigma)-2}\right) = 0$, we have $\pi(\sigma) =$

$$\begin{aligned}
& \sigma_1\Psi_1 + \cdot + \sigma_{k(\sigma)-1}\Pi_{j=1}^{k(\sigma)-2}\left(1-\sigma_j\right)\left[\sum_{j\geq k(\sigma)} \alpha_j\left(f^{k(\sigma)}\right) f(j)\left\{\mathbb{E}[\theta|j] - \left(k(\sigma)-1\right)\eta\right\}\right]. \\
& +\Pi_{j=1}^{k(\sigma)-1}\left(1-\sigma_j\right)\left[\sum_{j\geq k(\sigma)} \alpha_j\left(f^{k(\sigma)}\right) f(j)\left\{\mathbb{E}[\theta|j] - k(\sigma)\,\eta\right\}\right],
\end{aligned}$$

where α satisfies $E1$ under σ, and $\Psi_i = \sum_{j\geq i} \alpha_j\left(f^i\right) f(j)\left(\mathbb{E}[\theta|j] - i\eta\right)$. Think of the another commitment $\sigma' = (\sigma_1, ., \sigma_{k(\sigma)-2}, \delta/V, 1, ., \sigma_\xi)$. Now we have $\pi(\sigma') \geq$

$$\sigma_1\Psi_1 + \cdot + \delta/V\Pi_{j=1}^{k(\sigma)-2}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)-1}\alpha_j\left(f^{k(\sigma)-1}\right)f(j)\{\mathbb{E}[\theta|j]-(k(\sigma)-1)\eta\}\right].$$

$$+(1-\delta/V)\,\Pi_{j=1}^{k(\sigma)-2}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)}\alpha_j\left(f^{k(\sigma)}\right)f(j)\{\mathbb{E}[\theta|j]-k(\sigma)\eta\}\right],$$

because α satisfies $D1$ even under σ'. Obviously, we have $\pi(\sigma') > \pi(\sigma)$. Applying the same reasoning inductively backward, we can prove that for all commitment σ such that $\sigma_j < \delta/V$ for some j, there is a commitment σ' such that $\sigma'_j = \delta/V$ for all j and attains higher expected payoff for the DM.

Second, suppose that $\sigma_{k(\sigma)-1} > \delta/V$. Because $\alpha_j\left(f, f^{k(\sigma)-2}\right) = 1$, for all $j \geq k(\sigma)-1$, we have $\pi(\sigma) =$

$$\sigma_1\Psi_1 + \cdot + \sigma_{k(\sigma)-1}\Pi_{j=1}^{k(\sigma)-2}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)-1}f(j)\{\mathbb{E}[\theta|j]-(k(\sigma)-1)\eta\}\right].$$

$$+\Pi_{j=1}^{k(\sigma)-1}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)}f(j)\{\mathbb{E}[\theta|j]-k(\sigma)\eta\}\right],$$

where α satisfies $D1$ under σ, . Think of commitment $\sigma' = (\sigma_1, .., \sigma_{k(\sigma)-2}, \delta/V, 1, .., 1)$. Now the sender's strategy α' such that $\alpha'_j\left(f, f^{k(\sigma)-1}\right) = 0$ and $\alpha' = \alpha$ satisfies $D1$ under σ' and hence $\pi(\sigma) \geq$

$$\sigma_1\Psi_1 + \cdot + \delta/V\Pi_{j=1}^{k(\sigma)-2}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)}f(j)\{\mathbb{E}[\theta|j]-(k(\sigma)-1)\eta\}\right].$$

$$+(1-\delta/V)\,\Pi_{j=1}^{k(\sigma)-2}(1-\sigma_j)\left[\sum_{j\geq k(\sigma)}\alpha_j\left(f^{k(\sigma)}\right)f(j)\{\mathbb{E}[\theta|j]-k(\sigma)\eta\}\right],$$

which is strictly higher than $\pi(\sigma)$, because of (17). Applying the same reasoning inductively, we can prove that for all commitment σ such that $\sigma_j > \delta/V$ for some j, there is a commitment σ' such that $\sigma'_j = \delta/V$ for all j and attains higher expected payoff for the DM. *Q.E.D.*

9 Appendix B: Discussion on the best equilibrium

We can introduce an assumption, in addition to Assumption 1, that makes the characterization of the best equilibrium simpler. Towards this end, let ζ be the largest $j < \xi$ such that $\mathbb{E}[\theta|j] + \eta_S < 0$. Approximately, ζ is the sender type that the DM does not dare to pay the communication cost to screen it out. Obviously, for any equilibrium e, $N_f(e) < \zeta$.

Think of the function $\Gamma(j)$, defined in the main text. The assumption we want to impose is function Γ being decreasing. The function $\Gamma(j)$ is made by multiplying the loss from accepting type j sender's proposal with the probability that the sender's type being j relative to the probability that the sender type is strictly higher than j. Approximately, high $\Gamma(j)$ implies that the DM has strong incentive to screen out type j sender, after having already screened out lower types. A sufficient condition for this is that $|\mathbb{E}[\theta|j]|$ decreases fast enough to compensate for the change in the term $f(j) / \sum_{k \geq j+1} f(k)$, which is likely to be increasing. If $f(j) / \sum_{k \geq j+1} f(k)$ is increasing with j, the assumption is automatically satisfied.

To see that the assumption makes it easier to find the maximum length of persuasion, see condition (9). Under this condition, the maximum length of persuasion is determined by the largest κ such that $\eta < \Gamma(\kappa - 1)$. From the condition, we know that $\eta < \Gamma(l)$ for all $l < \kappa$ and this implies that we can find dropping vectors which can take only values less than one, in such a way that (7) is satisfied at each period. If the condition is not satisfied, the fact that j is the largest number satisfying $\eta < \Gamma(\kappa - 1)$ does not necessarily imply that the maximum length of persuasion is κ. To see this, think of the case that the assumption of Γ decreasing is not satisfied and $\eta < \Gamma(\kappa - 1)$ but $\eta > \Gamma(\kappa - 2)$. To be an equilibrium with maximum length of persuasion of κ, we must have (9) holding in order to support period $\kappa - 1$'s behavior of the DM (mixing between accepting and continuing) and we also have (11) for $t = \kappa - 2$, in order to support the DM's period $\kappa - 2$'s behavior. Here, note that only type $\kappa - 2$ or $\kappa - 1$ sender can drop at period $\kappa - 1$. However, if $\eta > \Gamma(\kappa - 2)$, it may not possible to choose d in such a way that (11) is satisfied for $t = \kappa - 2$, which implies that the maximum length of persuasion should be shorter than $\kappa - 2$.

In sum, under the condition of $\Gamma(j)$ being decreasing, the maximum length of persuasion is determined by j that satisfies

$$\Gamma(j) > \eta > \Gamma(j+1). \tag{18}$$

The assumption also makes it easier to find out the optimal dropping vector. As we discussed above, in finding the best equilibrium, we should use lower type sender's dropping to tie the consecutive period by the equality (11). In the equilibrium characterized in the theorem, we use only type j sender's dropping to make period j's equation (11). Apparently, type j is the lowest possible type to drop at period t. If Γ is decreasing, it is ensured that once we can make the equation (11) at period j satisfied by letting only type j sender drops at period j, it is also possible to make the equations hold at previous periods in the same way.

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